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EXISTENCE OF PASCAL'S TRIANGLE IN AN EXPONENTIAL FUNCTION

Fatima Farrukh[1*], Dania Tahir[1]
[1]Department of Mathematics, Kinnaird College for Women, Lahore.

Article Info

*Corresponding Author
Email:
fatima.farrukh@kinnaird.edu.pk

Abstract

In this paper, it is to be shown that Pascal's triangle exists in the function $f(n) = (n+1)^k j^{n+1}$ where $k \geq 0$ and $j \geq 2$ by using diagonal of diagonal approach applied in a forward difference table.

Keywords
Pascal's triangle, Diagonal approach, Arithmetical Triangle

1. Introduction

"A function f from a set X to a set Y is a rule or a correspondence that assigns to each element x in X a unique element y in Y. The set X is called the **domain** of f". The set of corresponding elements y in Y is called the **range** of f. But here we use an exponential function $f(n) = (n+1)^k j^{n+1}$. It is a type of function in which the functional value is raised to the power of the argument. Exponential function is the mathematical function of the form

$$f(x) = a^x$$

where x is defined as a variable and a as a constant known as the base of the function.

In 17*th* century, Pascal's triangle was named for the French mathematician Blaise Pascal, he used the matrix triangle in probability theory as part of his studies. He wrote the Treatise based on *Arithmetical Triangle* by using the already known array of binomial coefficients which is called Pascal's Triangle (Statisticshowto.com).

*row*0:					1				
*row*1:				1		1			
*row*2:			1		2		1		
*row*3:		1		3		3		1	
*row*4:	1		4		6		4		1

Figure 1. *Pascal's Triangle with four rows* (Sites.google.com)

Difference table is an auxiliary table to facilitate interpolation between numbers of the principal table, giving approximate differences in values of the tabulated function, corresponding to certain sub-multiples (such as tenths) of the constant smallest increment of the independent variable in the table (Merriam-webster.com). The formula we use to calculate forward difference is:

$$\Delta^r y_n = \Delta^{r-1} y_{n+1} - \Delta^{r-1} y_n$$

2. Material and Methods

Create a difference table by giving different values to the function $f(n) = (n+1)^k\, j^{n+1}$. Separate the terms present in the diagonal and create Pascal's triangle. Check whether Pascal's triangle is contained in the coefficients of diagonal terms obtained from the Forward Difference table, such as first term of diagonal is obtained from the first row of Pascal's triangle, second term is from the second row, third term from the third row of Pascal's triangle and so on.

3. Results

3.1. Theorem

Let f be a function on the non-negative integers with

$$f(n) = (n + 1)^k\, j^{n+1}.$$

Show that Pascal's triangle exists in this function.

3.2. Proof

Consider a function $f(n) = (n + 1)^k\, j^{n+1}$. Now to show that pascal's triangle exists in a function we have to construct forward difference table of this function.

n	$f(n)$	$\Delta f(n)$	$\Delta^2 f(n)$	$\Delta^3 f(n)$
0	$1^k j^1$			
		$2^k j^2 - 1^k j^1$		
1	$2^k j^2$		$3^k j^3 - 2(2^k j^2) + 1^k j^1$	
		$3^k j^3 - 2^k j^2$		$4^k j^4 - 3(3^k j^3) + 3(2^k j^2) - 1^k j^1$ 2
	$3^k j^3$		$4^k j^4 - 2(3^k j^3) + 2^k j^2$	
		$4^k j^4 - 3^k j^3$		
3	$4^k j^4$			

Table 1: FDT of *f* (n)

It is shown that the coefficients of the diagonal terms are determined by Pascal's triangle. Hence, we can say that Pascal's triangle exists in this function (Tahir, 2019).

4. Discussion

Numerical analysis has been the center of attention for many years and many researchers have worked on its techniques. The researcher generated forward difference table and analyzed the pattern of Pascal's triangle to determine the diagonal terms of forward difference table. Through this process the researcher concluded that Pascal's triangle exists in this function.

References

Auxiliary table, Retrieved from https://www.merriam-webster.com/dictionary/difference

Definition of Pascal's triangle, Retrieved from http://www.statisticshowto.com/pascals-triangle/

Pattern in Pascal's triangle, Retrieved from https://sites.google.com/site/mathematiciansactivity/bibliography/pascal-s-triangle-patterns

Tahir, D. (2019).Solution of an exponential function using Pascal's triangle and an alternate approach to exponential function. Paper presented at the Kinnaird's 1st International Conference on Natural and Applied Sciences, Lahore.