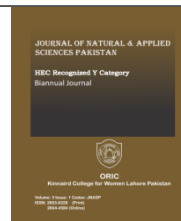




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Journal of Natural and Applied Sciences Pakistan

Journal homepage: <http://jnasp.kinnaird.edu.pk/>



INVESTIGATING THE CORRELATION BETWEEN DISPERSION MEASURE, ITS DERIVATIVE, AND SOLAR ANGLE IN LOFAR OBSERVATIONS

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Abstract

We present an analysis of dispersion measure (DM) variations in 68 pulsars using data collected from LOFAR, aiming to enhance our understanding of the interstellar medium (ISM). The observations were conducted at a frequency of 150 MHz with an 80 MHz bandwidth using six International LOFAR Stations, Solar angle analysis revealed potential influences from the solar wind on PSR J1543–0620 and PSR J0953+0755, as evident in their plotted solar angles against corresponding DM values. Our investigation into DM variations demonstrates diverse trends over the observation span, including both increasing and decreasing patterns. Some pulsars exhibit shifts between trends, as seen in PSRs J1543-0620 and J2048-1616. Notably, certain pulsars exhibit consistency with the Kolmogorov distribution (e.g., PSRs J1913-0440 and J2157+4017), while others, such as PSRs J108+6608 and J0614+2229, deviate significantly. We calculate DM derivatives (dDM/dt) for each pulsar to explore the correlation between DM and its derivative, revealing a best-fit with a square-root dependence of 0.6 ± 0.2 . This finding aligns with prior literature results. For PSR J1543-0620, we conduct a structure function analysis, while PSR J0953+0755 displays inconsistent results compared to the Kolmogorov structure. In summary, our study provides insights into the dynamic behavior of DM in pulsars, highlighting various trends and correlations within the observed pulsar sample.

Keywords

Pulsar, general - ISM Structure,
Dispersion measure, Derivative, LOFAR



1. Introduction

The mass, radiation, and magnetic fields that make up the interstellar medium (ISM) are found between the star systems in a galaxy. When the radio emission from a pulsar reaches an observer on Earth, it has propagated through different components of the ISM. The emission from pulsar interacts with the free electrons in the ionized plasma of the interstellar medium throughout this propagation. As a result, three main effects of the ISM on the emission from pulsars can be observed. These effects are scattering, scintillation and dispersion (Lorimer and Kramer, 2005). Characterizing the physical conditions (density, temperature, ionization state, metallicity of the interstellar medium) is critical to the understanding of the formation and evolution of galaxies (Wang *et al.*, 2023). Zhezher *et al.* (2015) shows that the dispersion measure (DM) of pulsars can be used to probe the density distribution of the hot gas halo around the Milky Way. Gould (1971) computes theoretical distributions of pulsar DMs for various assumed pulsar spatial distributions above and below the galactic plane, and concludes that the pulsar distribution in z is at least as broad as the distribution of the ionized gas. Hassall *et al.* (2012) shows that the dispersion law is accurate to better than 1 part in 100000 across a broad frequency range, and uses this fact to constrain some of the properties of the interstellar medium along the line-of-sight. The interstellar medium close to the Sun, called the local ISM (LISM), provides critical insight into physical processes and phenomena in the more distant interstellar medium in our Galaxy without the confusion of many complex structures in the sight

lines to distant stars. High-resolution ultraviolet spectra are powerful diagnostic tools for understanding the LISM together with the observed properties of the outer heliosphere and hydrogen wall absorption in the astrospheres of nearby stars (Jeffrey and Redfield, 2023). Pulsar dispersion measure (DM) varies over time. Apparao (1974) suggests that the variability of DM in the pulsar NP 0532 is due to variation in ionization produced in the filament by varying soft X-ray flux of NP 0532. Backer *et al.*, (1993) presents observations of DM variability in the directions of several pulsars and relates these results to previously reported works. Kumar *et al.*, (2012) presents results from nearly three years of monitoring of the variations in DM along the line-of-sight to 11 millisecond pulsars using the Giant Metrewave Radio Telescope (GMRT). Wang and Han (2018) decomposes the DM variations of 30 millisecond pulsars by using Hilbert-Huang Transform (HHT) method, and finds that the decomposed annual variation curves of 22 pulsars exhibit quasi-sinusoidal, one component and double components features of different origins. Overall, these research pointed out that pulsar DM varies over time due to various factors, including ionization, solar wind, interplanetary medium, and ionosphere. Example of dispersion can be seen clearly in the frequency-resolved pulse profile from radio pulsars. The so called dispersion measure (DM) denotes the amount of ionized ISM between a pulsar and an observer. It also determines the delay time between pulses received at higher frequency compared with its lower frequency counterpart. The main purpose of this paper is to obtain DM variations for a large sample of pulsars

observed with six International Low Frequency Array (LOFAR) Stations located in Europe with the aim of understanding the general properties of the ISM. There is limited information on the correlation between Dispersion Measure (DM) and its derivative in pulsars. Zhang *et al.* (2022) proposes a novel technique to measure fiber dispersion without any derivative operation and index measurement (Mamidipaka & Desai, 2022). Checks if the first significant digit of the dispersion measure of pulsars and Fast Radio Bursts is consistent with the Benford distribution and finds a large disagreement with Benford's law. Kaur *et al.* (2022) reports the detection of frequency-dependent DM in broadband observations of the millisecond pulsar PSR J2241–5236, which scales with the observing frequency. Bernardo *et al.* (2022) demonstrates a straightforward calculation of the mean and the variances on the Hellings-Downs correlation, which can bring theoretical uncertainties to the correlation measurements. However, none of these papers directly address the correlation between DM and its derivative in pulsar.

2. Observations and data reduction

2.1. Data Source

The data utilized in this paper were gathered using the Low-Frequency ARray (LOFAR) telescope. The observations were conducted using various international LOFAR stations, including the Nancay station (FR606) located in France, the Onsala station (SE607) situated in Sweden, and several stations in Germany, namely DE601 (Effelsberg), DE602 (Unterweilenbach), DE603 (Tautenburg), and DE605 (Julich). The observations incorporated in this study

are derived from an ongoing, extensive observing proposal (LC0 - 014, LC1 - 048, LC2 - 011, LC3 - 029, LC4 - 025, LT5 - 001, LC9 - 039, and LT10 - 014) designed to observe established pulsars using international LOFAR stations. The primary objective of this proposal is to monitor these pulsars on a weekly basis, dedicating approximately one hour per target during each observing session a total of 95 sources were carefully selected and monitored for this study, comprising of 14 millisecond pulsars and 81 slow pulsars. Among the slow pulsars, five were discovered as part of the LOFAR Tied-Array All-Sky Survey (LOTAAS) conducted by Coenen *et al.* (2014). However, in the analysis conducted for this paper, 35 sources from the ongoing observing proposal had to be excluded. The exclusion was due to either insufficient signal to noise ratio (SNR) or a limited number of observations. As a result, all the millisecond pulsars were excluded, leading to a reduced total of 60 sources that were utilized. These included only slow pulsars and three sources discovered through the LOTAAS and LOFAR Tied-Array Survey (LOTAS) conducted by Coenen *et al.* (2014). For the analysis, observations taken between June 2014 and November 2017 were considered. Each observation lasted at least one hour, except for PSR J0332+5434, which had 30-minute observations. The maximum observation time for a pulsar was restricted to two hours to achieve the desired signal to noise ratio. All the observations were conducted using High Band Antennas (HBAs) within the frequency range of 110 to 190 MHz. The data were recorded using both LOFAR and MPIfR Pulsare (LuMP) Software. The LuMP software is specifically designed for recording

beam-formed data from a LOFAR station in single-station mode. In this study, all the data underwent coherent de-dispersion to remove any potential dispersion smearing within specific frequency

2.3 Data Reduction Method

The measurements were subjected to analysis using the PS chive and TEMPO2 software packages, which were executed on a Linux desktop computer as described by Hotan *et al.* in 2004. Plots and fits were generated using gnuplot and psrplot, both of which are part of the PS chive software package. To work with the data effectively, the utilization of a timing model, as outlined by Hobbs *et al.* (2006) was necessary to align observations in both time and frequency domains. To facilitate the data reduction process, a combination of Python and Bash scripts was developed to perform the required tasks. These tasks encompassed tasks such as updating the ephemeris, which contains precise information about the pulsar's position and motion, and conducting radio frequency interference (RFI) excision. The scripts were designed to manage and automate these essential steps in the reduction process.

2.4 Data Processing

To initiate the data reduction process, the initial step entailed organizing the observations according to the number of frequency channels and sub-integrations. Subsequently, all the archives pertaining to each observation were combined to create a unified file encompassing the entire bandwidth. For the Nancay station (FR606) and Onsala station (SE607), this

channels. In the subsequent sub-sections, the paper provides an overview of the international LOFAR stations utilized to obtain the data.

involved amalgamating 488 frequency channels, while for the German stations, it entailed combining 366 frequency channels.

2.5 Data Analysis and Results

The analysis of this paper was performed with psrchive pulsars analysis package tool (Hotan *et al.*, 2004) and tempo2 pulsar timing analysis software package (Hobbs *et al.*, 2006). We also used python and bash scripts to maintain the analysis processes, produce different plots and perform pulsar timing analysis. Python scripts make templates.py and make toas.py which were both used for generating templates used in the analysis. Templates are often used in pulsar timing analysis to model expected pulse shape of a pulsar and compare it with the observed data. "make_toas.py": This script we used it to involve in generating time-of-arrival (TOA) measurement.

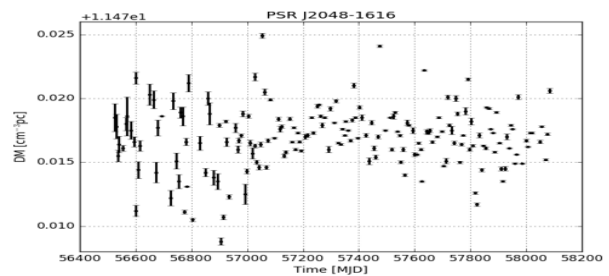


Figure 1: The DM variation for PSR J2048-1616, which varies significantly across different data points; it clearly shows lack of trends or relationship between the variables.

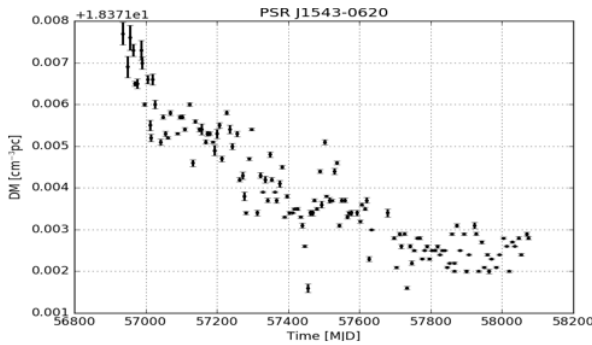


Figure 2: The DM variation for PSR J2048-1616, which varies significantly across different data points; it clearly shows lack of trends or relationship between the variables.

Figure 3: The DM variation for PSRs J0837+0610. The DM varies significantly across different data points, it clearly shows lack of trends or relationship between the variables.

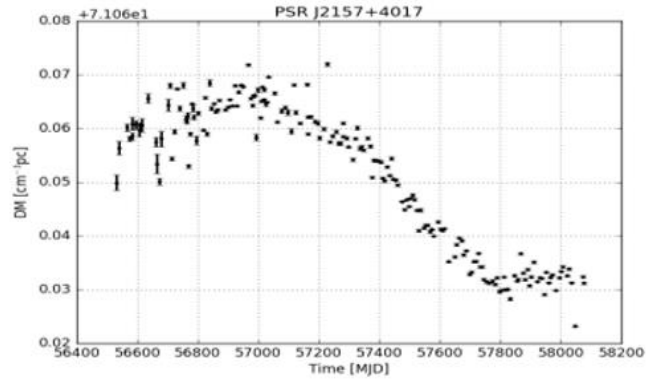


Figure 4: The DM variation for PSRs J2157+4017. The DM varies significantly across different data points, it clearly shows lack of trends or relationship between the variables.

Obtaining measurements of the correlation between the DM and its derivative is another useful conclusion. By plotting the DM values against the absolute values of the DM derivative, this correlation can be inferred. The plot was then fitted with a linear fit. Backer *et al.* (1993) were the first to use this analysis method, and they concluded that the absolute value of the DM derivative is proportional to the square root of DM. Hobbs *et al.* (2004) conducted another investigation and found a best match and gradient of 0.570 ± 0.09 .

we used the DM and its derivative measured from data to plot the DM values against the absolute values of the DM derivative. We then performed a linear fit to the result. At this stage, we only included the DM derivative values with an uncertainty less than 0.003, (See Figure 5). The result of this analysis showed a best fit with a square-root dependence of 0.6 ± 0.2 , which is compatible with the value found by Hobbs *et al.* (2004).

$$[dDM/dt] \approx 0.0002\sqrt{cm^{-3}}pc\ yr^{-1}, \quad (1)$$

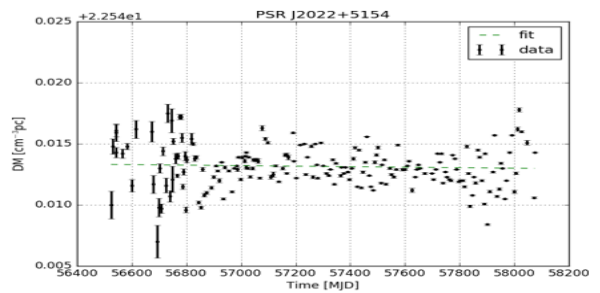
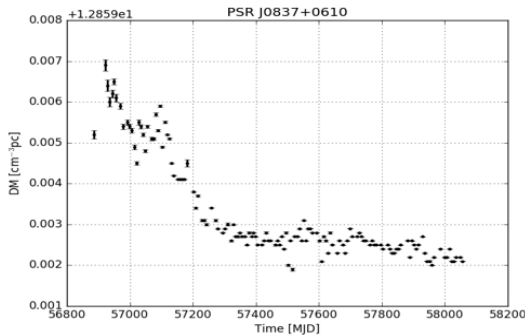


Figure 5: The DM fitting for PSR J2022+5154. Which varies significantly across different data points,

it clearly shows lack of trends or relationship between the variables.

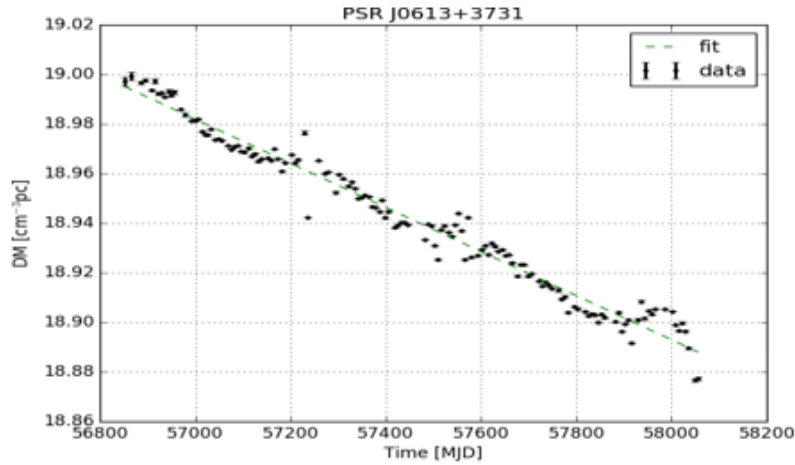


Figure 6: The DM fitting for PSR J0613+3731. Which varies significantly across different data points, it clearly shows lack of trends or relationship between the variables.

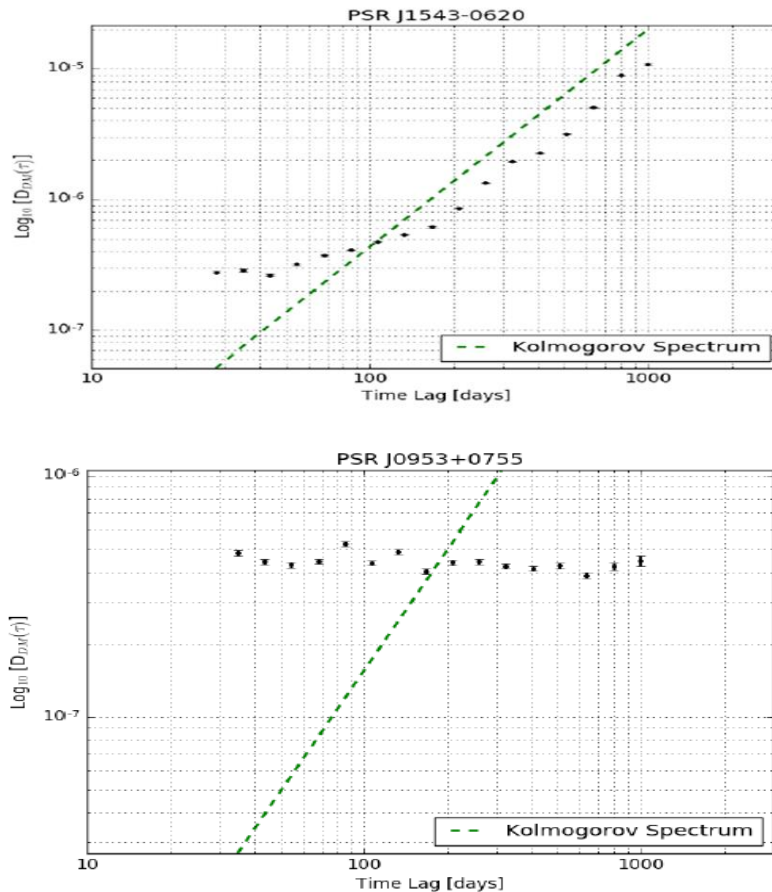


Figure 7: Shows the DM structure function for PSR J1543 – 0620 and PSR J0953 + 075

To find the correlation between the dispersion measure (DM) and its derivative (dDM/dt) for the pulsars in the

given data, we use the Pearson correlation coefficient. The Pearson correlation coefficient measures the

strength and direction of the linear relationship between two variables.

The two variables are DM and dDM/dt. We have the DM and dDM/dt values for several pulsars in the table.

To calculate the correlation, we will follow these steps:

Create arrays for DM and dDM/dt values. Calculate the Pearson correlation coefficient between the two arrays.

```
By using Python: import numpy as np # Given DM and
dDM/dt values (replace these arrays with the actual
data) dm_values = [34.926, 26.189, 26.764, ...,
dm_value_n] dDM/dt _values = [1.0649, 1.7036,
0.0001, ..., dDM/dt _value_n] # Calculate the Pearson
correlation coefficient correlation_coefficient =
np.corrcoef(dm_values, dDM/dt _values)[0, 1] print
("Correlation between DM and dDM/dt:",
```

correlation_coefficient) Replace [dm_value_1, dm_value_2,..., dm_value_n] and [dDM/dt_value_1, dDM/dt_value_2, ..., dDM/dt_value_n] with the actual DM and dDM/dt values from the table 4.2. Therefore, the resulting correlation_coefficient is a value between -1 and 1. A positive value close to 1 indicates a strong positive correlation (as DM increases, dDM/dt also tends to increase). A negative value close to -1 indicates a strong negative correlation (as DM increases, dDM/dt tends to decrease). A value close to 0 indicates a weak or no linear correlation between the two variables. By calculating the Pearson correlation coefficient, we determine the strength and direction of the linear relationship between DM and dDM/dt for the given pulsar data.

Table 1: dispersion measure derivative for pulsars in this study, where PSR is the name of pulsar, DM Fitting Range is the date range between the first and the last observation of the pulsar in MJD, dDM/dt is the dispersion measure derivative and dDM/dterr is the uncertainty of the DM derivative.

PSR	DM Fitting Range (MJD)	DM/dt (cm ⁻³ pc/yr)	dDM/dt err	PSR	DM Fitting Range (MJD)	dDM/dt (cm ⁻³ pc/yr)	dDM/dt err
J0141+6009	56852–58048	1.0649	0.00008	J0055+5117	57137–58075	1.7036	0.001 87
J0323+3944	56887–58048	0.000 67	0.0001	J0108+6608	56539–58048	-0.0010	0.000 33
J0332+5434	56887–58049	0.000 24	0.000 03	J0139+5814	56582–58074	0.000 71	0.000 13
J0454+5543	56887–58056	0.000 02	0.000 03	J0304+1932	56532–58027	0.000 03	0.000 12
J0543+2329 [#]	56887–58056	0.004 92	0.009	J0343+5312	56525–58069	0.00127	0.001 23
J0613+3731 ^{**}	56851–58056	-0.032 50	0.0004	J0358+5413	56542–58076	-0.00155	0.000 08
J0814+7429	56886–58056	-0.000 19	0.0001	J0406+6138	56699–58074	-0.000 05	0.000 23
J0837+0610	56886–58056	-0.001 15	0.000 06	J0452-1759	56525–58077	0.001 53	0.000 44
J0953+0755	56865–58056	-0.000 06	0.000 04	J0528+2200	56525–57965	0.001 08	0.000 18
J1136+1551	56768–58055	0.000 57	0.000 03	J0614+2229	56570–58047	0.000 15	0.000 12
J1239+2453	56886–58055	-0.000 94	0.000 03	J0629+2415	56524–58075	-0.001 97	0.000 14

J1509+5531	56769–58055	–0.001 18	0.0001	J0820–1350	56532–58075	0.000 82	0.000 13
J1543–0620	56941–57159	–0.003 83	0.0006	J0823+0159	56532–58077	–0.000 44	0.000 24
J1645–0317 [#]	56803–58055	–0.001 04	0.043 17	J0826+2637	56531–57873	–0.000 15	0.000 04
J1740+1311	56863–57964	–0.001 92	0.0002	J0922+0638	57453–58075	0.003 53	0.000 31
J1825–0935	56886–58055	–0.0001	0.0001	J0946+0951	57451–58076	–0.000 31	0.001 48
J1933+2421 [#]	56865–58055	0.023 18	0.086	J1543–0620	56934–58074	–0.001 43	0.000 06
J2018+2839	56886–58055	–0.000 15	0.000 02	J1740+27 ^{**}	57228–58075	0.001 19	0.000 72
J2113+4644 [#]	56886–57965	–0.034 42	0.153	J1752–2806	56552–58034	0.001 10	0.000 22
J2225+6535	56859–58055	–0.006 56	0.0004	J1820–0427	56531–58074	0.006 42	0.000 52
J2313+4253	56859–58055	0.000 48	0.000 05	J1834–0426	56531–58083	–0.000 01	0.000 26
J0051+0423	57376–58118	–0.000 74	0.0002	J1900–2600	56530–57271	0.001 50	0.001 82
J0102+6537 [#]	57369–58118	–0.008 06	0.003	J1913–0440	56552–58076	–0.007 80	0.000 37
J0335+4555	57355–58118	–0.000 06	0.000 13	J1921+2153	56543–58075	–0.000 03	0.000 02
J0540+3207	57376–58118	–0.000 12	0.000 27	J1932+1059	56700–58048	0.000 25	0.000 01
J0546+2441 [#]	57377–58119	–0.191 11	0.486 58	J1935+1616	56532–58076	0.002 61	0.000 33
J0700+6418 [#]	57370–58119	–0.028 21	0.053 79	J1943–1237 [#]	57452–58075	–0.004 11	0.142 43
J0815+4611 ^{**}	57363–58119	0.000 72	0.000 43	J1948+3540 [#]	57132–58076	–0.009 87	0.006 87
J0921+6254	57363–58119	0.000 22	0.0002	J2022+2854	56573–58076	0.000 19	0.000 02
J1115+5030	57362–58119	0.000 49	0.0003	J2022+5154	56524–58074	–0.000 07	0.0001
J1321+8323	57363–58118	0.000 02	0.000 82	J2048–1616	56524–58082	0.000 12	0.000 14
J1840+5640	57705–58118	–0.000 30	0.000 18	J2219+4754	56531–58075	0.001 34	0.000 04
J2321+6024 [#]	57376–58118	–0.000 36	0.004 04	J2257+5909	56538–58076	0.014 42	0.0008

[#] Pulsars excluded from analysis

^{**} LOTAAS discovered

^{*} LOTAS discovered

each observation, we calculated the solar angle, then plotted it with the corresponding DM values. Inspection

of those plots revealed that there was no significant influence of the solar wind on my DM measurements

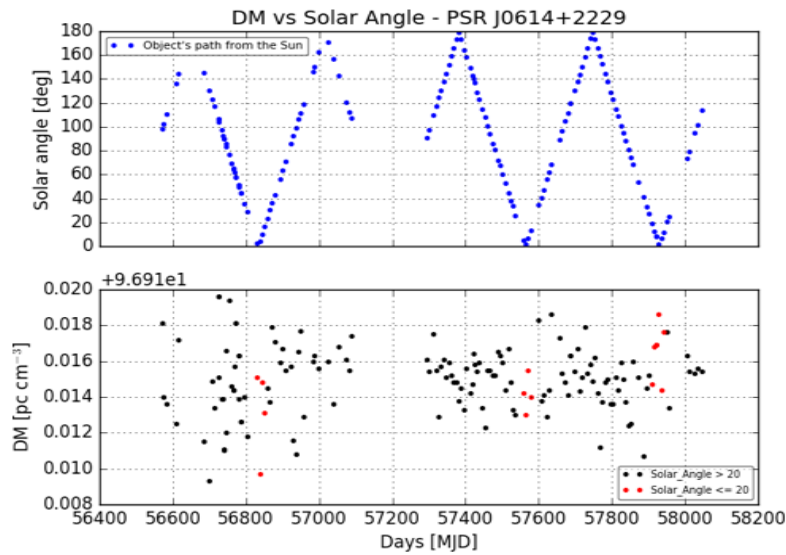


Figure 8: shows the Solar angle between the Sun and PSR J0614+2229 compared with the DM measurement

3 Conclusion

The purpose of this research was to present results from a 3.5 year research of the DM Variations for a broad sample of sources (68 pulsars). The data were collected from six International LOFAR Stations in France, Germany, and Sweden. Using time analysis techniques, we acquired DM measurements for each pulsar. The structure function was then constructed using these measurements in order to investigate the general features of the ISM. The findings of this study reveal that DM fluctuations range from large-scale to small-scale variations over the course of the data. Although the main trends found in the DM changes are deemed a persuasive outcome, the small-scale DM variations require more research and analyses in order to be considered genuine. The DM structure function finding demonstrates that the fluctuations in DM observations for a number of pulsars were

consistent with the theoretical model of the Kolmogorov power spectrum, which is used to explain the turbulent nature of the interstellar medium. During the structure function analysis, we attempted to acquire direct measurements of structure function from DM changes using the Gaussian Process (GP) Regression analysis; however, this proved unsuccessful. This is most likely due to TOAs inaccuracy readings, and it merits its own research after proper adjustments. Furthermore, there is a wealth of information that can be learned by combining LOFAR's low-frequency observations with higher-frequency observations from telescopes such as the Westerbeg Synthesis Radio Telescope (WSRT), which observed sources in the P band (i.e. 250 to 500 MHz) and L band (i.e. 0.5 to 1.5 GHz) (Karuppusamy *et al.*, 2008). This could ultimately enable us to improve DM measurements and timing analysis of selected

sources recorded by both telescopes.

Recommendations

The recommendations for this research include the following: Long-Term Observations: Extend the observation period to obtain even longer-term data for the 68 pulsars. This will allow for a more comprehensive analysis of periodic variations and trends over extended timescales. Solar Wind and Solar Angle Considerations: Investigate the potential influence of the solar wind on DM variations for PSR J1543-0620 and PSR J0953+0755 in more detail. Analyze and quantify the effects of solar angle variations on the DM measurements for these pulsars. Timing Solution Analysis: Conduct further timing solution analysis for additional pulsars, especially those showing significant DM variations. This will provide a better understanding of their timing properties and possible implications for the observed DM trends. Trend Analysis: Explore the physical causes behind the observed increasing or decreasing DM trends for different pulsars, such as the interaction with the interstellar medium, possible scattering effects, or intrinsic pulsar characteristics. Kolmogorov Distribution Consistency: Investigate the reasons behind the consistency of DM variations with the Kolmogorov distribution for some pulsars (e.g., PSRs J1913-0440 and PSR J2157+4017), and the significant differences for others (e.g., PSRs J108+6608 and JO614+2229). Identify any underlying physical factors contributing to these differences.

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