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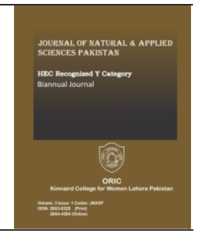

FUNDAMENTAL PROPERTIES OF SEE TRANSFORM AND ITS DUALITIES

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Abstract

In this paper, the fundamental properties of SEE transform are proposed after an in depth analysis of the Integral transforms and their properties. SEE transform is used to find the solutions of both ordinary and partial differential equations. The application of the properties derived in this paper would make the solution of differential equations less complex. Furthermore, relations between SEE transform and some already existing transforms were established.

1. Introduction

An integral transform in mathematics is a concept that represents a function in one domain to another domain via integration, where the properties of the given or original function are more easily described and doable than in the original domain or function space (Deakin, 1985). Because of their importance, the process of developing new integral transforms started and from the two most famous transforms called the Fourier and Laplace transforms, many other transforms were generated like Sumudu (Belgacem *et al*., 2006), Swai, Mohand and Elzaki transform etc. SEE transform is used in solving linear differential equations. Multiplying by t property and Change of Scale property of SEE integral transform were proposed in this research work as these properties of integral transforms are widely used in solving problems in linear differential equations.

1.1 SEE Transform

An integral transform developed by Sadiq, Emad A.Kuffi, Eman M. called SEE transform works with differential equations in time domain. It is closely associated with Laplace, Aboodh, Maghoub and Mohand transform. For given set A,

$A=\{f(t):\ \exists M,l\_1,l\_2>0,|f(t)|<Me^{(l\_i\ |t|\ )},\text{if}\ \ t\in(-1)^i\times[0,\infty)\}$

Where M is the finite number and can be finite or infinite. The SEE transform denoted by S (.) is given by:

$$S[f(t)] = T(v) = \frac{1}{v^n} \int_0^\infty f(t)\, e^{-vt}\, dt, \quad n \in Z, t \geq 0 \qquad (1.1.1)$$

Where the variable v factors the variable t in argument of function f(t). This integral transform has many applications physics, engineering, bio medical signal processing and is used for function with exponential order (Mansour *et al*., 2021).

1.2 Mohand Transform

For a given function f(t), the Mohand transform (Aggarwal *et al*., 2019) is defined as

$$M[f(t)] = M(v) = v^2 \int_0^\infty f(t)\, e^{-vt}\, dt \qquad (1.2.1)$$

1.3 Kamal Transform

For a given function f(t) provided the existence of integral, the Kamal transform (Aggarwal *et al*, 2019) is defined as

$$K[f(t)] = K(v) = \int_0^\infty f(t)\, e^{\frac{-t}{v}}\, dt$$

1.4 Laplace Transform

Let f(t) be a function of time then provided the existence of integral, the Laplace transform 0f f(t) is given by

$$L[f(t)] = L(v) = \int_0^\infty f(t)\, e^{-tv}\, dt$$

Laplace transform is one of the most famous integral transform.

1.5 Sumudu Transform

Let f(t) be a function of time, then provided the existence of integral, the Sumudu transform is as following for $f(t), t \geq 0$

$$S[f(t)] = S(v) = \int_0^\infty f(tv)\, e^{-t}\, dt$$

Sumudu transform is one of the easiest and widely used transform.

1.6 Mohand Transform

For a given function f(t), the Mohand transform is given as

$$M[f(t)] = M(v) = v^2 \int_0^\infty f(t)\, e^{-vt}\, dt$$

1.7 Maghoub Transform

For a given function f(t), the Maghoub transform is given as

$$M[f(t)] = M(v) = v \int_0^\infty f(t)\, e^{-vt}\, dt$$

1.8 Elzaki Transform

For a given function f(t), the Elzaki transform is given as

$$E[f(t)] = E(v) = v^2 \int_0^\infty f(vt)\, e^{-t}\, dt \qquad (1.3.1)$$

2. Results

2.1 Dividing by t property of SEE transform

Let $f \in A$ and T(v) is the SEE transform of f(t), then

$$S\left[\frac{f(t)}{t}\right] = \frac{1}{v^n} \int_v^\infty v^n\, T(v)\, dv$$

Proof:

The SEE transform is as follows

$$S[f(t)] = T(v) = \frac{1}{v^n} \int_0^\infty f(t)\, e^{-vt}\, dt$$

Now,

$$v^n\, T(v) = \int_0^\infty f(t)\, e^{-vt}\, dt$$

Integrating both sides with respect to v with limits v to ∞

$$\int_0^\infty v^n\, T(v) = \int_v^\infty \int_0^\infty f(t)\, e^{-vt}\, dt\, dv$$

$$\int_0^\infty v^n\, T(v) = \int_0^\infty \int_v^\infty f(t)\, e^{-vt}\, dv\, dt$$

Continuing in this way, we get the result for dividing by t property as follows

$$S\left[\frac{f(t)}{t}\right]=\frac{1}{v^n}\int_v^\infty v^n T(v)dv$$

Hence proved.

2.2 SEE-Mohand Duality

Let $f\in A$ and the SEE transform and Mohand transform are $T(v)$ and$M(v)$, then

$$T(v)=\frac{1}{v^{n+2}}M(v)$$

and

$$M(v)=v^{n+2}T(v)$$

Proof:

The SEE transform from equation (1.1) is as follows

$$T(v)=\frac{1}{v^n}\int_{t=0}^\infty f(t)e^{-vt}dt$$

Using suitable realtions and multiplying with required variables, we get the following relations between SEE and Mohand transform

$$T(v)=\frac{1}{v^n}\frac{v^2}{v^2}\int_{t=0}^\infty f(t)e^{-vt}dt$$

$$T(v)=\frac{1}{v^{n+2}}M(v)$$

Similarly, continuing in this way from equation (1.2) we get:

$$M(v)=v^{n+2}T(v)$$

Hence proved.

2.3 SEE-Kamal Duality

Let $f\in A$ and the SEE transform and Kamal transform are $T(v)$ and $K(v)$, then

$T(v)=\frac{1}{v^{n+2}}K(v)$ And $K(v)=v^{n+2}T(v)$

Proof:

The SEE transform from equation (1.1) is given as:

$$T(v)=\frac{1}{v^n}\int_{t=0}^\infty f(t)e^{-vt}dt$$

$$T(v)=\frac{1}{v^n}K\left(\frac{1}{v}\right)$$

We know that,

$$K(v)=\int_0^\infty f(t)e^{\frac{-t}{v}}dt$$

Using suitable relations and multiplying with required variables, we get the following relation

$$K(v)=\frac{1}{v^n}T\left(\frac{1}{v}\right)$$

Hence proved.

2.4 SEE-Laplace Duality

Let $f\in A$ and the SEE transform and Laplace transform are $T(v)$ and $L(v)$. Then:

$$T(v)=\frac{1}{v^n}L(v)$$

$$L(v)=v^n T(v)$$

Proof:

The SEE transform from equation 1.1 is given as:

$$T(v)=\frac{1}{v^n}\int_0^\infty f(t)e^{-vt}dt$$

$$T(v)=\frac{1}{v^n}L(v)$$

Similarly, The Laplace transform is given as:

$$L[f(t)]=L(v)=\int_0^\infty f(t)e^{-vt}dt$$

$$L(v)=\frac{v^n}{v^n}\int_0^\infty f(t)e^{-vt}dt$$

$$L(v)=v^n T(v)$$

Hence proved.

2.5 SEE-Elzaki Duality

Let $f\in A$ and the SEE transform and Elzaki transform are $T(v)$ and $E(v)$. Then:

$$T(v)=\frac{1}{v^{n+1}}E\left(\frac{1}{v}\right)$$

$$E(v)=\frac{1}{v^{n+1}}T\left(\frac{1}{v}\right)$$

Proof:

The SEE transform from equation 1.1 is given as:

$$T(v)=\frac{1}{v^n}\int_0^\infty f(t)e^{-vt}dt$$

Let $t=\frac{w}{v}, dt=\frac{1}{v}dw$;

$$T(v)=\frac{1}{v^n}\int_0^\infty f\left(\frac{w}{v}\right)e^{-v\left(\frac{w}{v}\right)}\frac{1}{v}dw$$

$$T(v)=\frac{1}{v}\frac{1}{v^n}\int_0^\infty f\left(\frac{w}{v}\right)e^{-w}dw$$

$$T(v)=\frac{v}{v^n}E\left(\frac{1}{v}\right)$$

$$T(v)=\frac{1}{v^{n+1}}E\left(\frac{1}{v}\right)$$

Similarly, the Elzaki transform is given as

$$E[f(t)]=E(v)=v^2\int_0^\infty f(vt)e^{-t}dt$$

Let $w=tv, dw=vdt$;

$$E(v)=v^2\int_0^\infty f(w)\, e^{-w/v}\,\frac{1}{v}\,dw$$

$$E(v)=v\int_0^\infty f(w)\, e^{-w/v}\,dw$$

Simplifying the above equation, we get

$$E(v)=\frac{1}{v^{n+1}}\,T(1/v)$$

Hence proved.

2.6 SEE-Sawi Duality

Let $f\in A$ and the SEE transform and Sawi transform are $T(v)$ and $S(v)$. Then:

$$T(v)=\frac{1}{v^{n+2}}\,W(1/v)$$

$$W(v)=\frac{1}{v^{n+2}}\,T(1/v)$$

Proof:

The SEE transform from equation 1.1 is given as:

$$T(v)=\frac{1}{v^n}\int_0^\infty f(t)\, e^{-vt}\,dt$$

$$T(v)=\frac{v^2}{v^2}\,\frac{1}{v^n}\int_0^\infty f(t)\, e^{-vt}\,dt$$

$$T(v)=\frac{1}{v^{n+2}}\,W(1/v)$$

Similarly, the Sawi transform is given as

$$W(v)=\frac{1}{v^2}\int_0^\infty f(t)\, e^{-t/v}\,dt$$

$$W(v)=\frac{1}{v^2}\,\frac{v^n}{v^n}\int_0^\infty f(t)\, e^{-t/v}\,dt$$

$$W(v)=\frac{1}{v^{n+2}}\,v^n\int_0^\infty f(t)\, e^{-t/v}\,dt$$

$$W(v)=\frac{1}{v^{n+2}}\,S(1/v)$$

Hence proved.

2.7 SEE-Sumudu Duality

Let $f\in A$ and the SEE transform and Sumudu transform are $T(v)$ and $N(v)$. Then:

$$T(v)=\frac{1}{v^{n+1}}\,N(1/v)$$

$$N(v)=\frac{1}{v^{n+1}}\,T(1/v)$$

Proof:

The SEE transform from equation 1.1 is given as:

$$T(v)=\frac{1}{v^n}\int_0^\infty f(t)\, e^{-vt}\,dt$$

Let $t=w/v, dt=1/v\,dw$

$$T(v)=\frac{1}{v^n}\int_0^\infty f(w/v)\, e^{-v(w/v)}\,\frac{1}{v}\,dw$$

$$T(v)=\frac{1}{v}\,\frac{1}{v^n}\int_0^\infty f(w/v)\, e^{-w}\,dw$$

$$T(v)=\frac{1}{v^{n+1}}\,N(1/v)$$

Similarly, the Sumudu transform is given as

$$N(v)=\int_0^\infty f(tv)\, e^{-t}\,dt$$

Let $w=tv, t=w/v, dt=1/v\,dw$

$$N(v)=\frac{1}{v}\int_0^\infty f(w)\, e^{-w/v}\,dw$$

$$N(v)=\frac{v^n}{v^n\,v}\int_0^\infty f(w)\, e^{-w/v}\,dw$$

$$N(v)=\frac{1}{v^{n+1}}\,T(1/v)$$

Hence proved.

2.8 SEE-Maghoub Duality

Let $f\in A$ and the SEE transform and Maghoub transform are $T(v)$ and $H(v)$. Then:

$$T(v)=\frac{1}{v^{n+1}}\,H(1/v)$$

$$H(v)=\frac{1}{v^{n+1}}\,T(1/v)$$

Proof:

The SEE transform from equation 1.1 is given as:

$$T(v)=\frac{1}{v^n}\int_0^\infty f(t)\, e^{-vt}\,dt$$

$$T(v)=\frac{v}{v^{n+1}}\int_0^\infty f(t)\, e^{-vt}\,dt$$

$$T(v)=\frac{1}{v^{n+1}}\,H(v)$$

Similarly, for Maghoub transform

$$H(v)=v\int_0^\infty f(t)\, e^{-vt}\,dt$$

$$H(v)=v\,\frac{v^n}{v^n}\int_0^\infty f(t)\, e^{-vt}\,dt$$

$$H(v)=v^{n+1}\,T(v)$$

Hence proved.

2.9 Convolution property of SEE Transform

Let $f\in A$ and $S(v)$ is the SEE transform of $f(t)$, then

$$S[f(t)*g(t)]=v^n\,S[f(t)].S[g(t)]$$

Proof:

By definition of SEE transform, let

$$S[f(t)]=F(v)=\frac{1}{v^n}\int_0^\infty f(t)\, e^{-vt}\, dt$$

$$S[g(t)]=G(v)=\frac{1}{v^n}\int_0^\infty g(u)\, e^{-vu}\, du$$

$$F(v).G(v)=\frac{1}{v^n}\int_0^\infty f(t)\, e^{-vt}\, dt \,.\, \frac{1}{v^n}\int_0^\infty g(u)\, e^{-vu}\, du$$

$$F(v).G(v)=\frac{1}{v^{2n}}\int_0^\infty\int_0^\infty e^{-t+uv}\, dt f(t)g(u)dtdu$$

Let t=z-u, then dt=dz and applying Calculus

$$F(v).G(v)=\frac{1}{v^{2n}}\int_0^\infty\left[\int_u^\infty e^{-zv}\, f(z-u)g(u)dz\right]du$$

$$F(v).G(v)=\frac{1}{v^n}\, S\left[\int_0^z f(z-u)g(u)du\right]$$

$$F(v).G(v)=\frac{1}{v^n}\, S[f(t)*g(t)]$$

$$S[f(t)*g(t)]=v^n\, S[f(t)].S[g(t)]$$

Hence proved.

3. Discussion

The purpose of this research was to find the fundamental properties and dualities of the integral transform, namely the SEE integral transform. The properties and dualities were obtained after using integration techniques and understanding the nature of SEE transform respectively.

4. Conclusion

In this research work, the fundamental properties, namely the Change of Scale property and Multiplication by t property of SEE transform, SEE Mohand, SEE Kamal, SEE Maghoub, SEE Laplace, SEE Sumudu, SEE Sawi, SEE Elzaki dualities, Convulsion theorem were derived and proposed. These proposed properties of SEE transform can be used to solve linear ordinary and partial differential equations and can also be used in engineering sciences and mathematical physics.

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