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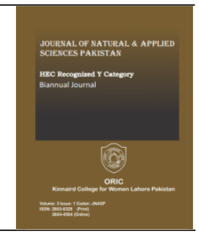

SOFTWARE RESULTS OF PURE RECURRENCE RELATION OF BATEMANS POLYNOMIAL AND EXTENDED BATEMAN POLYNOMIAL

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Abstract

In this research paper, results of pure recurrence relation of Bateman Polynomial ($Z_n(x)$) and Extended Bateman Polynomial ($S_n(x)$), which have already been derived by the researcher in a previous work, are rechecked using the software "MATLAB". This is done by using an inbuilt function within the MATLAB library. The augmented matrix of Bateman Polynomial and Extended Bateman Polynomial is run in MATLAB using the code “rref” that is reduced row echelon form. This technique is proposed to give us exact results.

1. Introduction

MATLAB is intended for technical computations, it helps the users to perform calculations within the blink of an eye. It aids in data analysis, mathematical computation, graphics, modeling, etc. Most of the commands are inbuilt functions within the MATLAB library that may be simply employed to perform the required functions. Thanks to its key options, MATLAB has been a part of many introductory courses in many universities majoring in the field of mathematics. In this research, a very easy and precise approach has been used for the Batemans and Extended Batemans polynomial. MATLAB (a mathematical software) has been used to find the values of unknowns (Aaboe, 1964). In this research paper, results of pure recurrence relation of Bateman Polynomial ($Z_n(x)$) and Extended Bateman Polynomial ($S_n(x)$), which have already been derived by the researcher in a previous work (Suhail, 2011).

2. Batemans Polynomial

Batemans polynomial that is $Z_n(x)$, is defined as

$$Z_n(x) = \;_2F_2(-n, n+1; 1, 1; x)$$

Batemans polynomial is being expanded by using the definition of Hypergeometric functions of type $_2F_2$ i.e. (E. D. Rainville)

$$Z_n(x) = \sum_{k=0}^{n} \frac{(-n)_k (n+1)_k}{(1)_k \left(\frac{1}{2}\right)_k} \frac{x^k}{k!}$$

$$Z_n(x) = \sum_{k=0}^{\infty} \varepsilon(k, n)$$

Sister Celine, in 1945 in her Ph.D. thesis (Murray, 1940), found a technique for the solution of pure recurrence relations for hypergeometric polynomials of the form $_2F_2$ (Zeilberger, 1982)

She showed her technique by obtaining pure recurrence relations for Batemans polynomial $Z_n(x)$ (Fasenmyar, 1949). In this research, Batemans Polynomial is being solved by an alternate technique i.e. by using Gaussian Elimination method to find values of unknowns using MATLAB in the following identity.

$$Z_n(x) + (A + B_x)Z_{n-1}(x) + (C + D_x)Z_{n-2}(x) + EZ_{n-3}(x) = 0$$

Now $Z_{n-1}(x)$, $Z_{n-2}(x)$, $Z_{n-3}(x)$, $xZ_{n-1}(x)$, $xZ_{n-2}(x)$ is expressed as the series involving $\varepsilon(k,n)$. Then substituting all these values in the identity (3) and reducing all coefficients of $\varepsilon(k,n)$ to the lowest common denominator, the equation becomes;

$$(n+k)(n+k-1)(n+k-2) + A(n-k)(n+k-1)(n+k-2) + B(-k^2)\left(\frac{1}{2}+k-1\right)(n+k-2)$$
$$+ C(n-k)(n-k-1)(n+k-2) + D(-k^2)\left(\frac{1}{2}+k-1\right)(n-k)$$
$$+ E(n-k)(n-k-1)(n-k-2) = 0$$

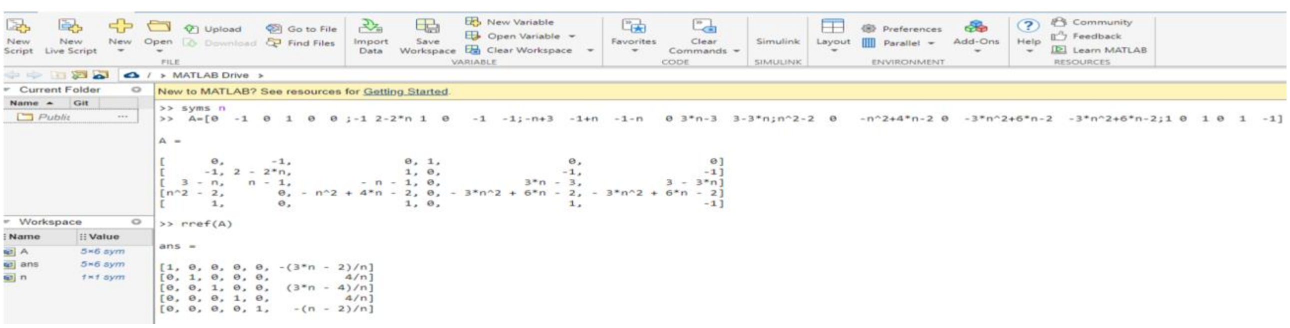

Figure 1: Results of Batemans polynomial

Simplifying the above equation and solving for constants A, B, C, D, and E, which are functions of n, not of k or x (Rainville, E. D.). Now solving the above identity in which highest power of k is four, then comparing coefficients of equal powers of k to zero, yields five equations. A system of equations is being solved by using Gaussian Elimination method (Anton,1987) considering the equation is linear in A, B, C, D, and E. We will get the values of unknowns from a matrix of order 5 by 6 by using an inbuilt function in MATLAB

By substituting the values of unknowns

$$A = \frac{-(3n-2)}{n}, B = \frac{4}{n}, C = \frac{3n-4}{n}, D = \frac{4}{n}, E = \frac{-(n-2)}{n}$$

Identity (3) becomes,

$$Z_n(x) + \left(\frac{-(3n-2)}{n} + \frac{4}{n}x\right) Z_{n-1}(x) + \left(\frac{3n-4}{n} + \frac{4}{n}x\right) Z_{n-2}(x) + \left(\frac{-(n-2)}{n}\right) Z_{n-3}(x) = 0$$

$$nZ_n(x) - (3n-2-4x)Z_{n-1}(x) + (3n-4+4x)Z_{n-2}(x) - (n-2)Z_{n-3}(x) = 0 \qquad)$$

Hence, pure recurrence relation for Bateman polynomial is obtained.

2. Extended Batemans Polynomial

Extended Batemans Polynomial that is $S_n(x)$, defined as (Suhail, 2012)

$$S_n(x) = \ {}_3F_3\left(-n, \frac{n+1}{2}, \frac{n+2}{2}; 1, 1, 1; x\right)$$

Expand this polynomial by using definition of Hypergeometric functions. Then further expand its terms using Pochhamer symbol and factorials $S_n(x) = \sum_{k=0}^{\infty} \frac{(-1)^k (n+2k)!}{(n-k)! 2^{2k}} \frac{x^k}{(k!)^4}$

Then rewrite $S_n(x)$ in the series involving $\delta(k,n)$.

$$S_n(x) = \sum_{k=0}^{\infty} \delta(k,n)$$

Extended Batemans Polynomial is being solved by the approach used for Batemans polynomial to find values of unknowns using MATLAB in the following identity (Suhail, 2012)

$$S_n(x) + (A + Bx)S_{n-1}(x) + (C + Dx)S_{n-2}(x) + (E + Fx)S_{n-3}(x) + GS_{n-4}(x) = 0$$

Now $S_n(x)$ for $S_{n-1}(x)$, $S_{n-2}(x)$, $S_{n-3}(x)$, $S_{n-4}(x)$, $xS_{n-1}(x)$, $xS_{n-2}(x)$, $xS_{n-2}(x)$ is expressed as the series involving $\delta(k,n)$, then substituting all these values in the identity (7) and reduce all coefficients of $\delta(k,n)$ to the lowest common denominator, the equation becomes

$$(n+2k-4)(n+2k-3)(n+2k-2)(n+2k-1)(n+2k)+A[(n-k)(n+2k-4)(n+2k-3)(n+2k-2)(n+2k-1)]+B[(-4k^4)(n+2k-4)(n+2k-3)]+C[(n-k)(n-k-1)(n+2k-4)(n+2k-3)(n+2k-2)]+D[(-4k^4)(n-k)(n+2k-4)]+E[(n-k)(n-k-1)(n+2k-4)(n+2k-3)]+F[(-4k^4)(n-k)(n-k-1)]+G[(n-k)(n-k-1)(n-k-2)(n-k-3)(n+2k-4)]=0$$

Simplifying and solving for constants A, B, C, D, E, F, and G which are functions of n, not of k or x. This identity leaves seven equations for determining the unknowns. Now solve the equation with 6, as the highest power of k, then equating coefficients of powers of k to zero. This yields seven equations. This system of equations is being solved by using the Gaussian Elimination method considering the equation is linear in A, B, C, D, E, F, and G. We will get the values of unknowns from a matrix of order 7 by 8. Similarly, the augmented matrix of Extended Bateman polynomial is run in MATLAB using code "rref". It will give exact values of unknowns A, B, C, D, E, F, G

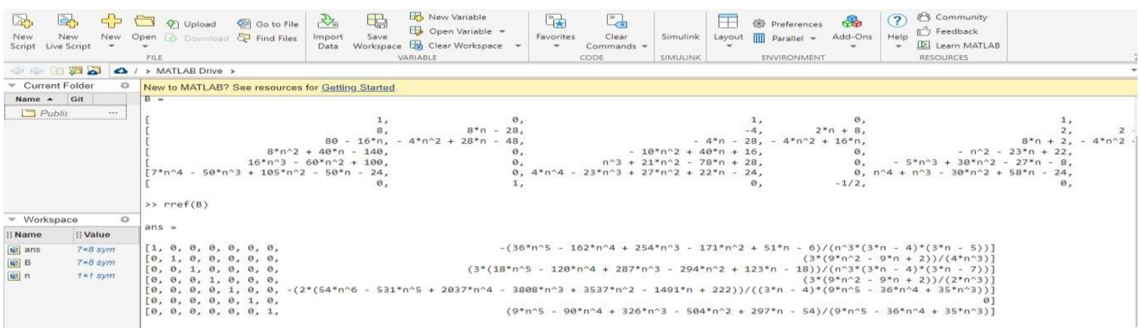

Figure 2: Results of Extended Batemans polynomial

By substituting the values of unknowns

$$A=\frac{-(36n^5-162n^4+254n^3-171n^2+51n-6)}{n^3(3n-4)(3n-5)}$$

$$B=\frac{3(9n^2-9n+2)}{4n^3}$$

$$C=\frac{3(18n^5-120n^4+287n^3-294n^2+123n-18)}{n^3(3n-4)(3n-7)}$$

$$D=\frac{3(9n^2-9n+2)}{4n^3}$$

$$E=\frac{-2(54n^6-531n^5+2037n^4-3808n^3+3537n^2-1491n+222)}{(3n-4)(9n^5-36n^4+35n^3)}$$

$$F = 0$$

$$G = \frac{(3n-1)(3n-2)(n-3)^3}{n^3(3n-5)(3n-7)}$$

Identity (7) becomes,

$$\begin{aligned}&\left(4n^3(3n-4)(3n-5)(3n-7)(9n^2-36n+35)\right)S_n(x)\\&+(3n-7)(9n^2-36n+35)(-4(36n^5-162n^4+254n^3-171n^2+51n-6)\\&+3(9n^2-9n+2)(3n-4)(3n-5)x)S_{n-1}(x)\\&+3(3n-7)(9n^2-36n+35)\Big((18n^5-120n^4+287n^3-294n^2+123n-18)\\&+2(9n^2-9n+2)(3n-4)(3n-5)x\Big)S_{n-2}(x)\\&-2(54n^6-531n^5+2037n^4-3808n^3+3537n^2-1491n+222)(3n-5)(3n\\&-7)S_{n-3}(x)+(3n-1)(3n-2)(n-3)^3(3n-4)(9n^2-36n+35)S_{n-4}(x)=0\end{aligned}$$

This is the solution of pure recurrence relation for Extended Batemans Polynomial.

4. Discussion

Special functions are the solutions to certain second-order differential equations. This research work provides a more precise and relatively error-free method to solve a pure recurrence relation.

5. Conclusion

In this research, Batemans polynomial and Extended Batemans polynomial has been solved using an alternate technique. This new technique allows the researcher to conclude that matrices are not confined only to linear algebra but can also be used to solve problems of other fields of mathematics, like in this paper a pure recurrence relation.

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