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**SOFT INTERSECTION ALMOST BI-INTERIOR IDEALS OF SEMIGROUPS**

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**Abstract**

As the notion of bi-interior ideal is a generalization of bi-ideal and interior ideal of a semigroup; soft intersection bi-interior ideal is a generalization of soft intersection bi-ideal and soft intersection interior ideal of a semigroup. In this study, we introduce the concept of soft intersection almost bi-interior ideal and its generalization soft intersection weakly almost bi-interior ideals of a semigroup. In contrast to the soft intersection ideal theory, we demonstrate that every soft intersection almost bi-interior ideal is a soft intersection almost bi-ideal and a soft intersection interior ideal. It is also illustrated that every idempotent soft intersection almost bi-interior ideal is a soft intersection almost subsemigroup, a soft intersection almost tri-ideal, and a soft intersection almost tri-bi-ideal. Furthermore, we derive a number of interesting relations concerning minimality, primeness, semiprimeness, and strongly primeness between almost bi-interior ideals and soft intersection almost bi-interior ideals with the obtained theorem that if a nonempty set A is almost bi-interior ideal, then its soft characteristic function is also soft intersection almost bi-interior ideal, and vice versa.



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1. **Introduction**

The early 1900s witnessed the formal study of semigroups. Semigroups are important in many mathematics disciplines since they give the abstract algebraic underpinning for "memoryless" systems, i.e., time-dependent systems that restart at each iteration. In practical mathematics, semigroups are fundamental models for linear time-invariant systems. The theory of finite semigroups has been critical, especially in theoretical computer science, as finite automata and finite semigroups are naturally connected. In probability theory, Markov processes are connected to semigroups.

To study algebraic structures and their uses, ideals are required. Dedekind first proposed ideals to help with the study of algebraic numbers. Noether then expanded on them by adding associative rings. Bi-ideals for semigroups were first developed by Good and Hughes (1952). The idea of quasi-ideals was initially put up by Steinfeld (1956) for semigroups, and subsequently for rings. Ideals in algebraic structures are to be generalized for further studies. In this regard, many mathematicians have made important contributions. Whereas the interior ideals are a generalization of ideals; the bi-ideals are a generalization of quasi-ideals.

Moreover, the notion of almost left, right, and two-sided ideals of semigroups was introduced by Grosek and Satko (1980). Later in 1981, Bogdanovic (1981) introduced the idea of almost bi-ideals in semigroups as a development of bi-ideals. In 2018, Wattanatripop, Chinram, and Changphas (2018a) introduced the notion of almost quasi-ideals of semigroups. In 2020, Kaopusek, Kaewnoi, and Chinram (2020) proposed the concepts of almost interior ideals and weakly almost interior ideals of semigroups and examined their characteristics using the notion of almost ideals and interior ideals of semigroups. The almost ideals of semigroups have attracted a lot of interest from researchers. Iampan, Chinram, and Petchkaew (2021), Chinram and Nakkhasen (2022), Gaketem (2022), and Gaketem and Chinram (2023) introduced the ideas of almost subsemigroups, almost bi-quasi-interior ideals, almost bi-interior ideals, and almost bi-quasi ideals of semigroups. Also, many types of almost fuzzy ideals of semigroups were investigated in Wattanatripop, Chinram, and Changphas (2018b), Iampan, Chinram, and Petchkaew (2021), Chinram and Nakkhasen (2022), Gaketem (2022), Gaketem, and Chinram (2023), Wattanatripop, Chinram, and Changphas (2018) and, Krailoet Simuen, Chinram, and Petchkaew (2021).

To model uncertainty, Molodtsov (1999) introduced the concept of a soft set. Since then, studies from a variety of domains have been interested in soft sets. Soft set operations, the foundation of the theory, were examined in by Maji, Biswas and Roy (2003), Pei and Miao (2005), Ali, Feng, Liu, Min and Shabir (2009), Sezgin and Atagün (2011), Feng, Jun and Zhao (2008), Ali, Shabir and Naz (2011), Sezgin, Ahmad and Mehmood (2019), Stojanovic (2021), Sezgin and Atagün (2023), Sezgin and Aybek (2023), Sezgin, Aybek and Atagün (2023a), Sezgin, Aybek and Güngör (2023b), Sezgin and Dagtotos (2023), Sezgin and Sarıalioğlu (2024), Sezgin and Çalışıcı (2024), Sezgin and Demirci (2023), Sezgin and Yavuz (2023a), Sezgin and Yavuz (2023b), Sezgin and Çağman (2024). The notion of a soft set and its operations were modified by Çağman and Enginoğlu (2010). Çağman, Çıtak, and Aktaş (2012) developed the idea of soft intersection groups, which prompted the research of several soft algebraic systems. Soft sets were used to apply semigroup theory by Sezer, Çağman, and Atagün (2014) and, Sezer, Çağman, Atagün, Ali, and Türkmen (2015a). Semigroups with soft intersection left (right/two-sided) ideals, quasi-ideals, interior ideals, and (generalized) bi-ideals were thoroughly examined by Sezer et al. (2014) and Sezer et al. (2015a). In terms of soft intersection substructures of semigroups, Sezgin and Orbay (2022) classified semisimple semigroups, duo semigroups, right (left) zero semigroups, right (left) simple semigroups, semilattice of left (right) simple semigroups, semilattice of left (right) groups, and semilattice of groups. A variety of algebraic structures, including soft sets, were also examined in Mahmood, Rehman, and Sezgin (2018), Jana, Pal, Karaaslan, and Sezgin (2019), Muştuoğlu, Sezgin, and Türk (2016), Tunçay and Sezgin (2016), Sezer and Atagün (2016), Sezer, Çağman, and Atagün (2015b), Sezer (2014a, 2014b), Sezgin (2016), Özlü and Sezgin (2020), Atagün and Sezgin (2018), Sezgin (2018), Sezgin, Çağman and Atagün (2017), Sezgin, Atagün, Çağman, and Demir (2022). As an extension of existing ideals, Rao (2018a, 2018b, 2020a, 2020b) has recently proposed a number of new types of semigroups, such as bi-interior ideals, bi-quasi ideals, bi-quasi-interior ideals, weak interior ideals, and quasi-interior ideals. Moreover, Baupradist, Chemat, Palanivel, and Chinram (2020) suggested the essential ideals of semigroups.

As a generalization of the bi-ideal and interior ideal, the bi-interior ideal of semigroups was proposed by Rao (2018a), and as a generalization of the soft intersection bi-ideal and the soft intersection interior ideal, the soft intersection bi-interior ideal of semigroups was proposed by Sezgin and İlgin (2024a, in press). In the present study, the notions of soft intersection almost bi-interior ideals and soft intersection weakly almost bi-interior ideals are presented. Our findings demonstrate that a soft intersection weakly almost bi-interior ideal of a semigroup is a soft intersection almost bi-interior ideal and contrary to soft intersection semigroup theory, every soft intersection almost bi-interior ideal is also a soft intersection almost bi-ideal and a soft intersection almost interior ideal; however, the converses are not true with counterexamples. Furthermore, we show that an idempotent soft intersection almost bi-interior ideal is a soft intersection almost subsemigroup, a soft intersection tri-ideal, and a soft intersection tri-bi-ideal. We note that a semigroup may be constructed by soft intersection almost bi-interior ideals of a semigroup under the binary operation of soft union, but not under the soft intersection operation. Also, by deriving that if a nonempty set A is almost bi-interior ideal, then its soft characteristic function is also soft intersection almost bi-interior ideal, and vice versa, we establish the relationship between a semigroup's soft intersection almost bi-interior ideal and almost bi-interior ideal as regards minimality, primeness, semiprimeness, and strongly primeness.

**2. Preliminaries**

In this section, we recall the fundamental notions related to semigroups and soft sets.

**Definition 2.1.** Let be the universal set, be the parameter set, be the power set of , and . A soft set over is a set-valued function such that such that for all , . A soft set over can be represented by the set of ordered pairs

(Molodtsov, 1999; Çağman and Enginoğlu, 2010). Throughout this paper, the set of all the soft sets over is designated by .

**Definition 2.2.** Let . If for all , then is called a null soft set and denoted by . If for all , then is called an absolute soft set, and denoted by (Çağman and Enginoğlu, 2010).

**Definition 2.3.** Let , . If for all then is a soft subset of and denoted by . If for all , then is called soft equal to and denoted by (Çağman and Enginoğlu, 2010).

**Definition 2.4.** Let, *.* The union of and is the soft set , where for all **.** The intersection of and is the soft set , where for all (Çağman and Enginoğlu, 2010).

**Definition 2.5.** For a soft set , the support of is defined by

 (Feng *et al.,* 2008).

It is obvious that a soft set with an empty support is a null soft set, otherwise, the soft set is nonnull.

**Note 2.6.** If, then (Sezgin and İlgin, 2024b).

A semigroup is a nonempty set with an associative binary operation, and throughout this paper, stands for a semigroup, and all the soft sets are the elements of unless otherwise specified.

**Definition 2.7.** A nonempty subset of is called,

1. a subsemigroup of if ,
2. a bi-ideal of if ,
3. an interior ideal of if ,
4. a bi-interior ideal of if ,
5. an almost bi-ideal of if , for all ,
6. an almost interior ideal of if , for all ,
7. a weakly almost interior ideal of if , for all ,
8. an almost bi-interior ideal (briefly A-bi-interior ideal) of if for all ,
9. a weakly almost bi-interior ideal (briefly weakly A-bi-interior ideal) of if for all .

A (weakly) almost bi-interior ideal of is called a minimal (weakly) almost bi-interior ideal of if for any (weakly) almost bi-interior ideal of if whenever , then . A (weakly) almost bi-interior ideal of is called a prime (weakly) almost bi-interior if for any (weakly) almost bi-interior ideals and of such that implies that or . A (weakly) almost bi-interior ideal of is called a semiprime (weakly) almost bi-interior ideal of if for any (weakly) almost bi-interior ideal of such that implies that . A (weakly) almost bi-interior ideal of is called a strongly prime (weakly) almost bi-interior ideal if for any (weakly) almost bi-interior ideals and of such that implies that or .

**Definition 2.8.** Let and be soft sets over the common universe . Then, soft intersection product is defined by (Sezer *et al.,* 2015a)

**Theorem 2.9.** Let, . Then,

1. .
2. and .
3. and .
4. If then and .
5. If *,*  such that and then (Sezer *et al*., 2015a).

It is obvious thator (Sezgin and İlgin, 2024c).

**Definition 2.10.** Let be a subset of . We denote by the soft characteristic function of and define as

The soft characteristic function of is a soft set over , that is,: (Sezer *et al.,* 2015a).

If for all , then we denote such a kind of soft set by throughout this paper. It is obvious that , that is, for all (Sezer *et al.,* 2015a).

**Corollary 2.11.**  (Sezgin and İlgin, 2024b).

**Theorem 2.12.** Let and be nonempty subsets of . Then, the following properties hold (Sezer *et al.,* 2015a; Sezgin and İlgin, 2024b):

1. if and only if
2. and

**Definition 2.13.** Let be an element in . We denote by the soft characteristic function of and define as

The soft characteristic function of is a soft set over , that is,: (Sezgin and İlgin, 2024c).

**Definition 2.14.** Asoft set of over is called

1. a soft intersection bi-interior ideal (briefly SI-bi-interior ideal) of over if (Sezgin and İlgin, 2024a, in press),
2. a soft intersection almost subsemigroup (briefly SI-A-subsemigroup) of over if (Sezgin and İlgin, 2024b),
3. a soft intersection almost bi-ideal (briefly SI-A-bi-ideal) of over if for all (Sezgin and Onur, 2024)
4. a soft intersection almost interior ideal (briefly SI-A-interior ideal) of over if for all (Sezgin and Baş, 2024, in press)
5. a soft intersection weakly almost interior ideal (briefly SI-weakly A-interior ideal) of over if for all (Sezgin and Baş, 2024, in press)
6. a soft intersection almost left (right) tri-ideal of over if for all and a soft intersection almost tri-ideal (briefly SI-A-tri-ideal) of if is both a soft intersection almost left tri-ideal of and a soft intersection almost right tri-ideal of (Sezgin, Onur and İlgin, 2024, in press)
7. a soft intersection almost tri-bi-ideal (briefly SI-A-tri-bi-ideal) of over if for all (Sezgin, İlgin and Atagün, 2024, in press)

It is easy to see that if for all , then is a soft intersection bi-interior ideal of . As is mentioned above, we denote such a kind of soft intersection bi-interior ideal by . Regarding the potential consequences of network analysis and graph applications on soft sets, which are determined by the divisibility of the determinant, we refer to (Pant, Dagtoros, Kholil and Vivas, 2024).

**III. Soft Intersection Almost Bi-interior Ideals of Semigroups**

**Definition 3.1.** A soft set is called a soft intersection almost bi-interior ideal of if

for all. is called a soft intersection weakly almost bi-interior ideal of if

for all .

Hereafter, for brevity, soft intersection almost bi-interior ideal and soft intersection weakly almost bi-interior ideal are denoted by SI-A-bi-interior ideal and SI-weakly A-bi-interior ideal**,** respectively. Here also note that since the operation of soft intersection is commutative in , it is obvious that in Definition 3.1, and can commute with each other for all.

**Example 3.2.** Let be the semigroup with the following Cayley Table.

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |

Let , and be soft sets over as follows:

Here, and are both SI-A-bi-interior ideals. Let’s first show that is an SI-A-bi-interior ideal, that is, , for all *.*

Let’s start with :

Consequently,

.

Similarly,

.

Therefore, for all, so is an SI-A-bi-interior ideal. Similarly, is an SI-A-bi-interior ideal. In fact;

.

Thus, , for all. Hence, is an SI-A-bi-interior ideal.

One can also show that is not an SI-A-bi-interior ideal. In fact;

Consequently,

Thus, is not an SI-A-bi-interior ideal.

Also here, and are both SI-weakly A-bi-interior ideals. Let’s first show that is an SI-weakly A-bi-interior ideal, that is, , for all:

Therefore, , for all. Thus, is an SI-weakly A-bi-interior ideal.

Similarly, is an SI-weakly A-bi-interior ideal. In fact;

Thus, , for all. Thereby, is an SI-weakly A-bi-interior ideal. One can also show that is not an SI-weakly A-bi-interior ideal. In fact;

Hence,

.

Thus, is not an SI-weakly A-bi-interior ideal.

**Proposition 3.3.** Every SI-A-bi-interior ideal is an SI-weakly A-bi-interior ideal.

**Proof:** Let be an SI-A-bi-interior ideal. Then,

.

for all . Hence, , for all . Thus, is an SI-weakly A-bi-interior ideal.

Since SI-weakly A-bi-interior ideal is a generalization of SI-A-bi-interior ideal, from now on all the theorems and proofs are given for SI-A-bi-interior ideal instead of SI-weakly A-bi-interior ideal.

The following example shows that the converse of Proposition 3.3 is not true in general:

**Example 3.4.** Consider the semigroup in Example 3.2. Define a soft set over such that .

By the usual calculations, it is obvious that is an SI-weakly A-bi-interior ideal, that is, , for all . Indeed,

Thus, is an SI-weakly A-bi-interior ideal. However, is not an SI-A-bi-interior ideal. In fact;

Therefore, is not an SI-A-bi-interior ideal. This completes the proof.

**Proposition 3.5.** Let be an SI-bi-interior ideal. Then, either for some , or is an SI-A-bi-interior ideal.

**Proof:** Let be an SI-bi-interior ideal thus and let . We need to show that

for allSince , it follows that .

From assumption is obvious. Then,

implying that is an SI-A-bi-interior ideal.

Here, it is obvious that is an SI-bi-interior ideal since

but is not an SI-A-bi-interior ideal as

**Corollary 3.6.** If is an SI-A-bi-interior ideal, then needs not be an SI-bi-interior ideal.

**Example 3.7.** In Example 3.2, it is shown that and are SI-A-bi-interior ideal; however, and are not SI-bi-interior ideals. In fact,

and so is not an SI-bi-interior ideal. Similarly,

Thus, is not an SI-bi-interior ideal.

**Theorem 3.8.** Every SI-A-bi-interior ideal is an SI-A-bi-ideal.

**Proof:** Assume that is an SI-A-bi-interior ideal. Hence,

for all . We need to show that

for all . Since it is obvious that . Hence, is an SI-A-bi-ideal.

The following example shows that the converse of Theorem 3.8 is not true in general:

**Example 3.9.** Consider the soft set in Example 3.4. It was shown in Example 3.4 that is not an SI-A-bi-interior ideal; however, is an SI-A-bi-ideal, that is, , for all . In fact,

Consequently, . Similarly,

Thus, is an SI-A-bi-ideal, although is not an SI-A-bi-interior ideal as seen in Example 3.4.

**Theorem 3.10.** Every SI-A-bi-interior ideal is an SI-A-interior ideal.

**Proof:** Assume that is an SI-A-bi-interior ideal. Hence,

for all . We need to show that , for all . As

it is obvious that . Hence, is an SI-almost interior ideal.

It is obvious that every SI-weakly A-bi-interior ideal is an SI-weakly A-interior ideal. The following example shows that the converse needs not to be true:

**Example 3.11.** Consider the soft set in Example 3.2. It was shown in Example 3.2 that is not an SI-weakly A-bi-interior ideal; however, here we show that is an SI-weakly A-interior ideal, that is, , for all Let’s start with :

Consequently, Similarly,

Thus, is an SI-weakly A-interior ideal; although is not an SI-weakly A-bi-interior ideal. Here, note that, since is not an SI-weakly A-bi-interior ideal, it is obvious that is not an SI-A-bi-interior ideal.

**Proposition 3.12.** If is an idempotent SI-A-bi-interior ideal, then is an SI-A-subsemigroup.

**Proof:** Assume that is an idempotent SI-A-bi-interior ideal. Then, and for all . We need to show that is an SI-A-subsemigroup, that is . Since,

and , it is obvious that . Thus, is an SI-A-subsemigroup.

**Proposition 3.13.** Let be an idempotent soft set. If is an SI-A-bi-interior ideal, then is an SI-A-tri-ideal.

**Proof:** Assume that is an SI-A-bi-interior ideal such that is an idempotent, then and for all . We need to show that is an SI-A-tri-ideal, that is and , for all . It is obvious that

Since , for all , it is clear that for all . Hence, is an SI-A-left tri-ideal. Similarly, it is obvious that

Since , for all , it is clear that for all . Hence, is an SI-A-right tri-ideal. Thus, is an SI-A-tri-ideal.

**Proposition 3.14.** Let be an idempotent soft set. If is an SI-A-bi-interior ideal, then is an SI-A-tri-bi-ideal.

**Proof:** Assume that is an SI-A-bi-interior ideal such that is an idempotent, then and for all . We need to show that is an SI-A-tri-bi-ideal, that is for all . As

and , it is obvious that . Thus, is an SI-A-tri-bi-ideal.

**Theorem 3.15.** If is an SI-A-bi-interior ideal, then is an SI-A-bi-interior ideal, where.

**Proof:** Assume that is an SI-A-bi-interior ideal. Hence,

for all .

We need to show that , for all . Indeed,

Since,

 it is obvious that . This completes the proof.

**Theorem 3.16.** Let and be SI-A-bi-interior ideal Then, is an SI-A-bi-interior ideal.

**Proof:** Since is an SI-A-bi-interior ideal and , is an SI-A-bi-interior ideal by Theorem 3.15.

**Corollary 3.17.** The finite union of SI-A-bi-interior ideals is an SI-A-bi-interior ideal.

**Corollary 3.18.** Let or be SI-A-bi-interior ideal, then is an SI-A-bi-interior ideal.

Here note that if and are SI-A-bi-interior ideal, then needs not be an SI-A-bi-interior ideal.

**Example 3.19.** Consider the SI-A-bi-interior ideal and in Example 3.2. Since,

 is not an SI-A-bi-interior ideal.

Now, we give the relationship between A-bi-interior ideal and SI-A-bi-interior ideal. But first of all, we give the following lemma in order to use it in Theorem 3.21.

**Lemma 3.20.** Let and be a nonempty subset of . Then, . If is a nonempty subset of and , then (Sezgin and İlgin, 2024c).

**Theorem 3.21.** Let be a nonempty subset of . Then, is an A-bi-interior ideal if and only if , the soft characteristic function of , is an SI-A-bi-interior ideal.

**Proof:** Assume that is an A-bi-interior ideal. Then, for all , and so there exist such that . Since,

it follows that . Thus, is an SI-A-bi-interior ideal.

Conversely assume that is an SI-A-bi-interior ideal. Hence, we have , for all . In order to show that is an A-bi-interior ideal, we should prove that and for all . is obvious from assumption. Now,

Hence, . Consequently, is an A-bi-interior ideal.

**Lemma 3.22.** Let . Then, (Sezgin and İlgin, 2024b)

**Theorem 3.23.** If is an SI-A-bi-interior ideal, then is an A-bi-interior ideal.

**Proof:** Assume that is an SI-A-bi-interior ideal. Thus, for all . In order to show that is an A-bi-interior ideal, by Theorem 3.21, it is enough to show that is an SI-A-bi-interior ideal. By Lemma 3.22,

and since

it implies that . Consequently, is an SI-A-bi-interior ideal and by Theorem 3.21, is an A-bi-interior ideal.

The following example shows that the converse of Theorem 3.23 is not true in general:

**Example 3.24.** Consider the soft set in Example 3.4. It was shown that is not an SI-A-bi-interior ideal as seen in Example 3.4. Since , it is obvious that , for all . That is to say, is an A-bi-interior ideal; although is not an SI-A-bi-interior ideal.

**Definition 3.25.** An SI-A-bi-interior ideal is called minimal if any SI-A-bi-interior ideal if whenever , then .

**Theorem 3.26.** Let be a nonempty subset of . Then, is a minimal A-bi-interior ideal if and only if , the soft characteristic function of , is a minimal SI-A-bi-interior ideal.

**Proof:** Assume that is a minimal A-bi-interior ideal. Thus, is an A-bi-interior ideal, and so is an SI-A-bi-interior ideal by Theorem 3.21. Let be an SI-A-bi-interior ideal such that . By Theorem 3.23, is an A-bi-interior ideal and by Note 2.6 and Corollary 2.11,

.

Since is a minimal A-bi-interior ideal, . Thus, is a minimal SI-A-bi-interior ideal by Definition 3.25.

Conversely, let be a minimal SI-A-bi-interior ideal. Thus, is an SI-A-bi-interior ideal and is an A-bi-interior ideal by Theorem 3.21. Let be an A-bi-interior ideal such that . By Theorem 3.21, is an SI-A-bi-interior ideal and by Theorem 2.12 (i), . Since is a minimal SI-A-bi-interior ideal,

by Corollary 2.11. Thus, is a minimal A-bi-interior ideal.

**Definition 3.27.** Let , , and be any SI-A-bi-interior ideals. If implies that or , then is called an SI-prime A-bi-interior ideal.

**Definition 3.28.** Let and be any SI-A-bi-interior ideals. If implies that , then is called an SI-semiprime A-bi-interior ideal.

**Definition 3.29.** Let , , and be any SI-A-bi-interior ideals. If implies that or , then is called an SI-strongly prime A-bi-interior ideal.

It is obvious that every SI-strongly prime A-bi-interior ideal is an SI-prime A-bi-interior ideal and every SI-prime A-bi-interior ideal is an SI-semiprime A-bi-interior ideal.

**Theorem 3.30.** If , the soft characteristic function of , is an SI-prime A-bi-interior ideal, then is a prime A-bi-interior ideal, where .

**Proof:** Assume that is an SI-prime A-bi-interior ideal. Thus, is an SI-A-bi-interior ideal and thus, is an A-bi-interior ideal by Theorem 3.21. Let and be A-bi-interior ideals such that . Thus, by Theorem 3.21, and are SI-A-bi-interior ideals, and by Theorem 2.12 (i) and (iii),

.

Since is an SI-prime A-bi-interior ideal and , it follows that or . Therefore, by Theorem 2.12 (i), or . Consequently, is a prime A-bi-interior ideal.

**Theorem 3.31.** If , the soft characteristic function of , is an SI-semiprime A-bi-interior ideal, then is a semiprime A-bi-interior ideal, where .

**Proof:** Assume that is an SI-semiprime A-bi-interior ideal. Thus, is an SI-A-bi-interior ideal and thus, is an A-bi-interior ideal by Theorem 3.21. Let be an A-bi-interior ideal such that . Thus, by Theorem 3.21, is an SI-A-bi-interior ideal and by Theorem 2.12 (i) and (ii),

.

Since is an SI-semiprime A-bi-interior ideal, and , it follows that . Therefore, by Theorem 2.12 (i) . Consequently, is a semiprime A-bi-interior ideal.

**Theorem 3.32.** If , the soft characteristic function of , is an SI-strongly prime A-bi-interior ideal, then is a strongly prime A-bi-interior ideal, where .

**Proof:** Assume that is an SI-strongly prime A-bi-interior ideal. Thus, is an SI-A-bi-interior ideal and thus, is an A-bi-interior ideal by Theorem 3.21. Let and be A-bi-interior ideal such that . Thus, by Theorem 3.21, and are SI-A-bi-interior ideals, and by Theorem 2.12,

Since is an SI-strongly prime A-bi-interior ideal and , it follows that or . Thus, by Theorem 2.12 (i), or . Therefore, is a strongly prime A-bi-interior ideal.

**4. Conclusion**

In this study, we defined the concepts of soft intersection almost bi-interior ideal and soft intersection weakly almost bi-interior ideal of semigroups. We showed that every soft intersection almost bi-interior ideal is a soft intersection weakly almost bi-interior ideal, a soft intersection almost bi-

ideal, and a soft intersection almost interior ideal of a semigroup; however, the converses are not true with counterexamples. Furthermore, it was demonstrated that an idempotent soft intersection almost bi-interior ideal is a soft intersection almost subsemigroup, a soft intersection tri-ideal, and a soft intersection tri-bi-ideal. With the obtained theorem that if a nonempty set A is almost bi-interior ideal, then its soft characteristic function is soft intersection almost bi-interior ideal, and vice versa, we obtained the relation between soft intersection

almost bi-interior ideal of a semigroup and almost bi-interior ideal of a semigroup according to minimality, primeness, semiprimeness, and strongly primeness. Furthermore, we explored that the binary operation of soft union can construct a semigroup with a collection of soft intersection almost bi-interior ideals, but not soft intersection operation. In the following studies, some types of soft intersection almost ideals, including bi-quasi ideals, quasi-interior ideals, and bi-quasi-interior ideals of semigroups may be examined.

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