TO STUDY THE EFFECTS OF DISTRIBUTION FUNCTION LINEAR AND NONLINEAR CORRELATIONS ON THE EXTREME EVENTS: HEART FAILURE DIAGNOSIS

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Abstract

In this paper, we first examine the effect of linear correlations and non-linear correlations between the data of the simulated series and also the effect of their distribution function on extreme events, we look at the R-R distance series (natural, extended series) and the effect of linear correlations and We examine non-linear correlations as well as the effect of their distribution function on the distribution of extreme values in them, and from the behavior of the distribution of their extreme values, we seek to find a significant difference between people without heart rhythm disorders and people with rhythm disorders. For this purpose, we calculate and plot the probability density function of the extreme values of these series using both the maximum block approach and beyond the threshold.

1. Introduction

Cardiac disorders are considered one of the most common causes of death in the world, and considering that their prevalence has an upward trend and the heavy costs associated with their treatment, research is very important to diagnose them to take treatment and care (Manshour, 2015).

Since humans have adapted to normal conditions, extreme events, which are rare events with an intensity far beyond normal, can have devastating effects on human society. In recent years, among researchers, interest in research on extreme events has increased significantly (Sharma et al., 2013) The

study of extreme events, as an interdisciplinary research field, includes sciences such as physics (Kishore et al., 2011), mathematics (Behrens et al., 2004), earth sciences (Moffatt & Shuckburgh, 2011), engineering (Alipour et al., 2013), economics (Diebold et al., 2000), and even sociology (Webb, 2002). Contains Events such as economic crises, floods and typhoons, earthquakes, droughts, and the like are examples of extreme events. In general, extreme value theory is used to determine extreme data in a sequence of data (time series), which provides two different methods for analyzing extreme data. In the first method, which is called the maximum blocks method, the maximum of the data is determined in consecutive periods (which for many experimental data are usually weekly, monthly or yearly), and these maximums are considered as extreme data. The second method of determining extreme data is called beyond the threshold method. In this method, any data that exceeds a threshold value is considered as an extreme data (Haan & Ferreira, 2006). One of the characteristics of extreme events is their rarity, so statistically, extreme events for any process are placed in the tails of its probability density function, and the values they accept are called extreme values. The origin of the extreme value theory goes back to the works of Fisher and Tippett (1928) and Gendenko (1943). So far, much research has been done in this field, which is available in the form of numerous articles and books (Kyselý et al., 2010). For example, in 1985, Jian Pencheng and his colleagues used extreme value theory to predict the probability of a superacute influenza outbreak in Zhejiang, China. In this study, using this theory, they investigated the periodical mode of influenza outbreaks related to two years (Fisher & Tippett, 1928).

2. Maximum Blocks Method

The maximum blocks method, which is also known as the Fisher-Tippet theorem, assumes that $x_1 \langle\!\langle ,x \rangle\!\rangle _2,\ldots,x_n$ are a sequence of independent data and the same distribution or iid for short, then these data are recorded in the same order ask are considered equal categories, so that in each category there is several m data, or in other words, our total number of observations includes n is the data that we divide into k blocks that include each block m data, as a result $n=m\times k$ Now, the maximum data corresponding to each block ith is considered an extreme value and is obtained according to the following equation(Albeverio et al., 2006).

$$x_i=\max\{x_{ik}\} \tag{1}$$

Where x_{ik} is the data set in the ith block out of the total number of k blocks? To obtain meaningful results, n must be a large number (which is assumed to be infinite in the theory) so that k and m are also large numbers compared to n. If we standardize the maximum of each block based on the mean and standard deviation of the same block, it is proven that the probability density function of the standardized maximums is one of the three Gamble, Weibel, and Freisht distributions, which are respectively expressed by the following relations (Kishore et al., 2011).

$$f(x)=1/\sigma \exp[-(x-\mu)/\sigma-\exp\{-(x-\mu)/\sigma\} \tag{2}$$

$$f(x)=c/b \langle\!\langle ((x-a)/b)\rangle\!\rangle ^{(c-1)} \exp\{-((x-a)/b)^c\} \tag{3}$$

$$f(x)=a/b \ [\![(b/(x-c))]\!] \ ^\wedge(a-1) \ \exp\{- \ [\![(b/(x-c))]\!]$$

$$^\wedge a\} \qquad (4)$$

These three distributions are known as extreme value distributions (Gencay & Selçuk, 2004). Figure (1) shows the blocking of an assumed time series and the selection of the maximum of each block.
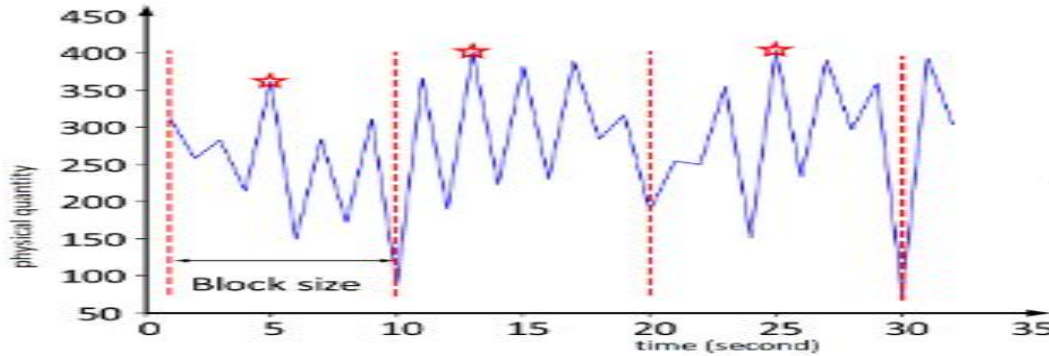

Figure 1: Shows The Blocking of Data

Figure 1 Shows The Blocking of Data and finding the maximum of each block in a hypothetical time series. The points marked with an asterisk show the maximum of each block, which is considered an extreme value.

3. Generalized Extreme Value Distribution

The generalized extreme value distribution was introduced in 1936 by von Mises. Von Mises showed that this distribution is considered a compressed form for the extreme values obtained from an iid distribution by the maximum block method. The general form of the probability density for the generalized extreme value distribution is expressed by the following equation (Rosso, 2016).

$$p(x)=1/\sigma \ [\![[1+\xi((x-\mu)/\sigma)]]\!] \ ^\wedge(-1/\xi-1) \ \exp\{[1+\xi((x-\mu)/\sigma)]^\wedge(-1/\xi)\} \qquad (5)$$

In the above relationship, ξ and μ can accept any real value and σ is a positive and real quantity, also the parameter ξ is called the extreme value index (Zhou et al., 2016).

4. Method Beyond the Threshold

In the early 1970s, Balkema, Hahn, and Pickends considered another method for statistical analysis of extreme values. According to this approach, which is called the beyond-threshold approach, all data that are greater than a threshold (which is usually predetermined) are considered as extreme values. Suppose $x\_1, x]\!]\_2, x\_n$ are time series data with iid properties, also suppose that u is a high enough threshold value for the extreme events of this time series, then the form of the probability density function of the data beyond this threshold u will be in the form of the following relationship(Kora & Kalva, 2015):

$$P(x)=\{ (1/\sigma+k \ [\![(((x-u))/\sigma)]\!] \ ^\wedge(-1-1/k)$$

$$k>0@1/\sigma \ \exp(-((x-u))/\sigma) \qquad k=0) \qquad (6)$$

In the above relationship, u is the threshold value. The parameter k can include any real value, while σ includes only real and positive numbers (Rosso, 2016). Figure (2) shows the extreme values of a time series for a threshold value equal to one(Manshour, 2015).
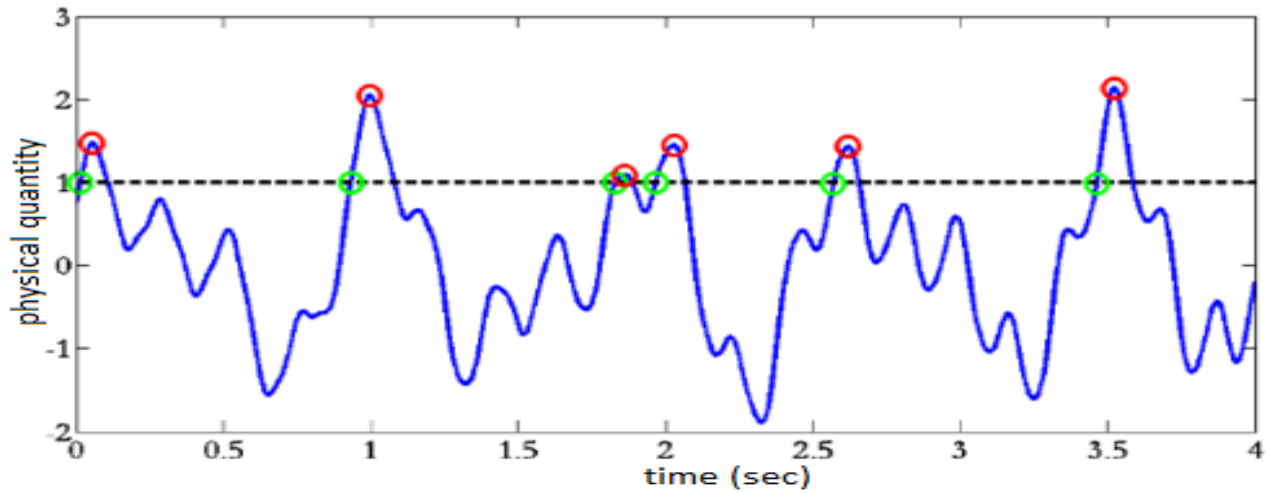

Figure 2: shows a time series

Figure 2 shows a time series for which the extreme value of the threshold is considered 1. All data that are equal to or greater than this threshold value (data on and above the dashed line) are considered extreme values.

5. Correlation and distribution function of fractal and multifractal series: Extreme Events

In this section, with the help of the MATLAB program, we simulate several fractal and multifractal time series and investigate the effect of three factors of linear and non-linear correlations and distribution function on the extreme events in them using We pay both maximum block and beyond threshold approaches. For this purpose, we produce 4 time series which include two fractal series fGn and fBm and two multi-fractal series, one of which is sign in size and the other is stable alpha. The produced series contains one million data, which first obtains the data of the extreme values of each by using the maximum block approach by

selecting blocks of size 1000 and then by going beyond the threshold by selecting a threshold equal to 2, and drawing their probability density function for 50 times of ensemble averaging. As the time series, fGn is a stationary series with Gaussian distribution and its correlation is only linear. The β parameter, which is the power spectrum view of the fGn series, has a relationship with the Hurst view of these series as follows:

$$\beta(H)=-1+2H \qquad (7)$$

Also, this parameter for fBm series is related to their Hurst view according to the following relationship [70, 71]:

$$\beta(H)=1+2H \qquad (8)$$

Since Hurst's view can express the strength of (linear) correlation between time series data, by changing the parameter β, the strength of linear correlation between the data of the series under study changes. In this way, we produce the fGn series in such a way that the strength of correlation

between the data by choosing β from -1 (which is the perfect negative correlation for the fGn series) to +1 (which is the perfect positive correlation for the fGn series) is) we change with steps of 0.4, then we draw the probability density function of its extreme values using the maximum block approach for different linear correlations, which can be seen in Figure (3) be As seen in this figure, the probability density distribution of extreme values is sensitive to the value of linear correlation between the data of a stable series with Gaussian distribution. It can be said that for stable series with Gaussian distribution, the probability density diagram of extreme values decreases with increasing β, and the maximum probability density of the distribution decreases. Now we go to the approach beyond the threshold and obtain the probability density functions corresponding to the mentioned βs according to this approach by choosing the threshold value equal to 2. The results of this approach are shown in Figure (4). In this figure, we can see that except for the case of β equal to -1, the probability density diagrams of extreme events are placed on top of each other, that is, in other correlations, they are not sensitive to the change of linear correlation. Also, the probability density of the threshold value for β=-1 (which is 2 here) is higher than other β values. Now we are going to produce the fBm fractal series, which is an unstable series with a Gaussian distribution and its correlation is only linear. It has +1 (complete negative correlation) to +3 (complete positive correlation) and is related to Hurst's view according to the relationshipβ=2H+1. Now we make an fBm series and change the strength of linear correlation

between its data from β=1.90 to β=3 with steps of 0.4, then the probability density function of its extreme values for linear correlations we draw differently, which can be seen in Figure (5). As β increases, the probability density distribution diagrams of extreme values move to the left, and in large β, they almost overlap. Now we go to the approach beyond the threshold and get the probability density functions corresponding to the mentioned βs according to this approach by choosing the threshold value equal to 2. The results of this approach are shown in Figure (6). In this figure, with the change of β, a general trend in the behavior of the probability density of extreme values cannot be reported. To investigate the effect of non-linear correlation, we create a multifractal series of signs in the size whose distribution is Gaussian, with a linear correlation value of zero, and increase the non-linear correlation β from -1 to 2 with steps of 0.5 Then we draw the probability density function of its extreme values using both maximum block and beyond the threshold approaches for different linear correlations. The graphs obtained from these two approaches can be seen in Figures (7) and (8). As we can see in these figures, with the change of β, a general trend in the behavior of the probability density of extreme values cannot be reported. Now let's examine the effect of the distribution function on the distribution of extreme events in the absence of any correlation, we pay for this purpose, we generate a stable alpha series and each time, we change the α profile that expresses the distance from Gaussianity. We increase the α view from 1.90 to 2, with steps of 0.02, where α=1.90 is the most distant from

Gaussianity and $\alpha = 2$ leads to Gaussian distribution. For each value of α, we get the probability density of extreme values on a logarithmic scale. In Figure (9), the probability density functions of extreme values in a logarithmic scale, obtained from the maximum block approach, are shown. In this figure, it can be seen that in stable alpha series, for the distribution function with α values less than 2, the behavior of the probability density of the extreme values is similar and no significant difference can be found between them. Also, the probability density functions of the extreme values in a logarithmic scale, obtained from the approach beyond the threshold for the threshold value equal to 2, are shown in Figure (10). In this figure, we can see that a general rule for the behavior of the probability density of the extreme values of the stable alpha series, for different views of α, cannot be presented beyond the threshold.
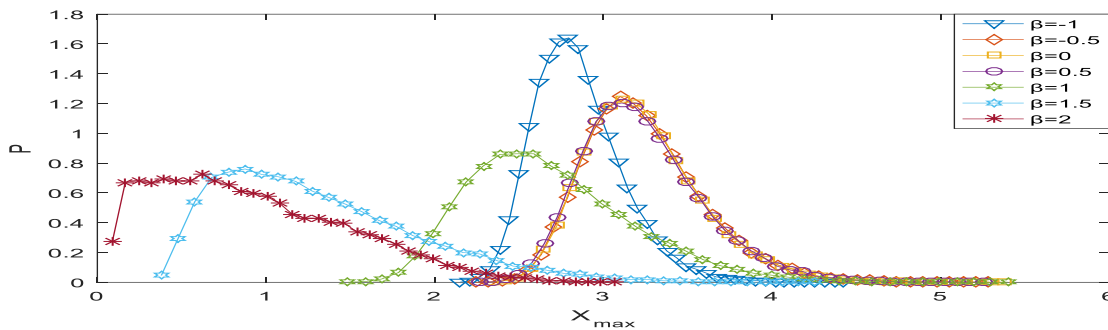
Figure 3: Graphs of the probability density functions of the extreme values obtained from the maximum block method for the time series fGn under study show different correlations.
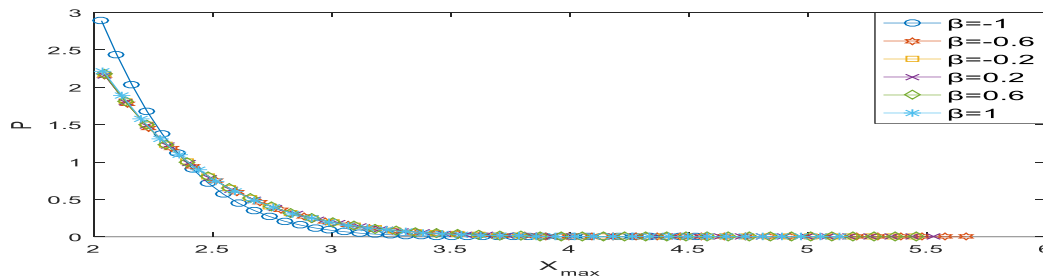

Figure 4: shows the graphs of the probability density functions of the extreme values obtained from the beyond-the-threshold method for the time series fGn under study, in different correlations
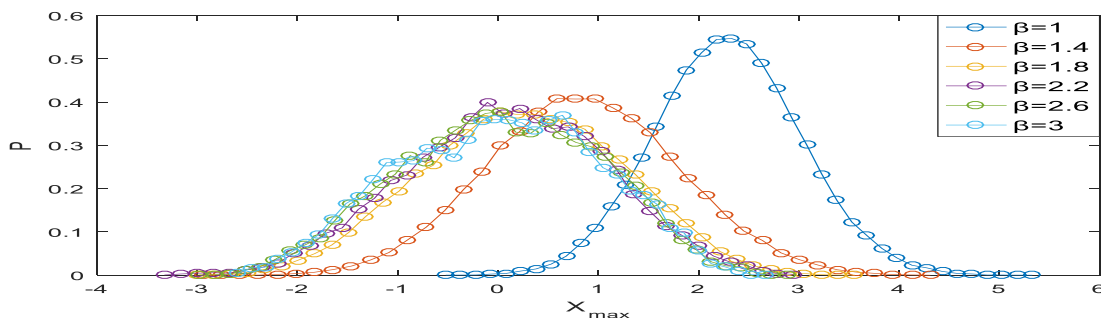
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Figure 5: above shows the graphs of the probability density functions of the extreme values obtained from the maximum block method for the fBm time series under study, in different correlations.
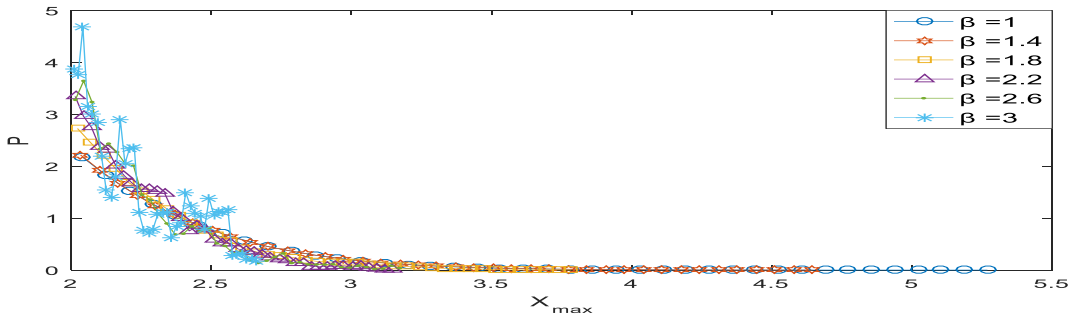
Figure 6: shows the graphs of the probability density functions of the extreme values obtained from the beyond-the-threshold method for the fBm time series under study, in different correlations.
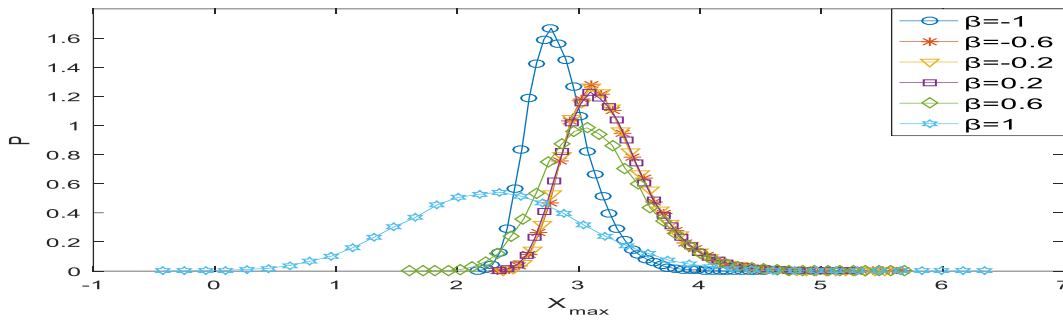

Figure 7: shows the graphs of the probability density functions of the extreme values obtained from the maximum block method for the time series of the sign in the size under study, in different correlations.
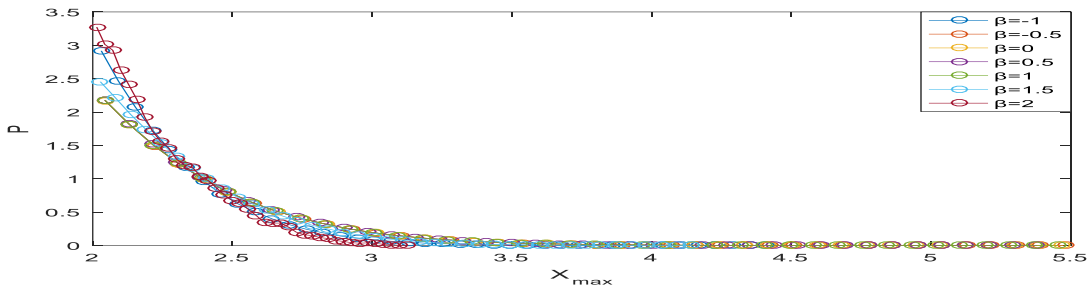

Figure 8: shows the graphs of the probability density functions of the extreme values obtained from the method beyond the threshold for the time series of the sign in the size under study, in different correlations.
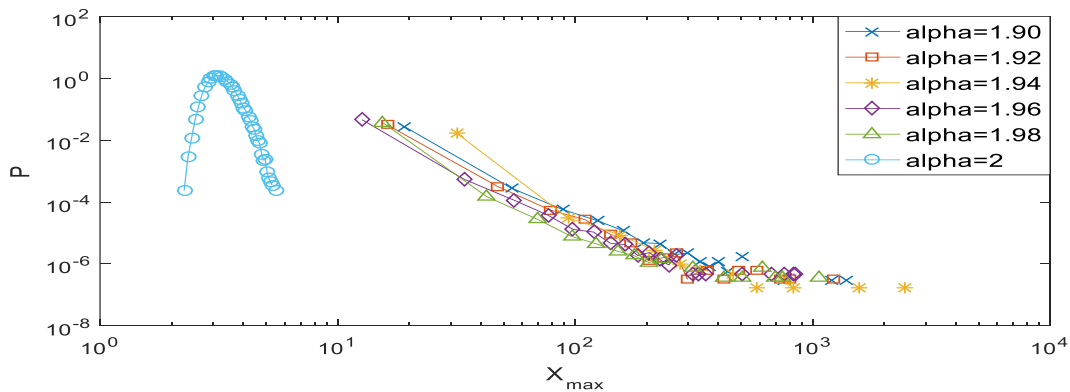
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Figure 9: shows the graphs of the probability density functions of the extreme values obtained from the maximum block method for a stable alpha time series, with the change of the α profile, in a logarithmic scale.
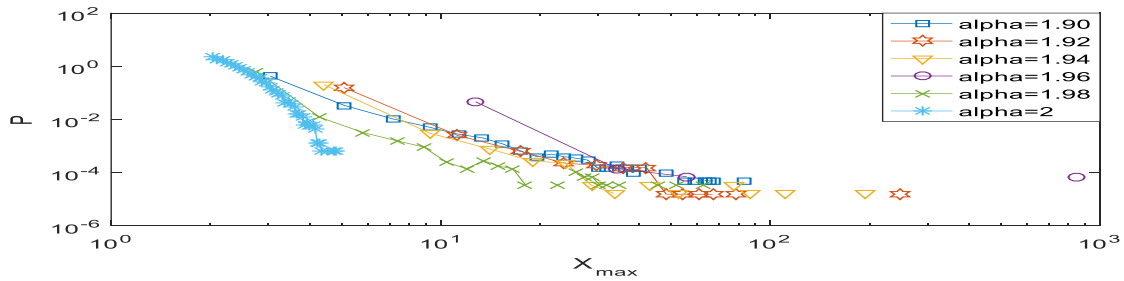
Figure 10: shows the graphs of the probability density

Figure (10) shows the graphs of the probability density functions of the extreme values obtained from the beyond-threshold method for a stable alpha time series, with the change of the α profile, in a logarithmic scale. To obtain the RR interval series, first, we calculate the interval of both consecutive R waves in the ECG chart and number each of these intervals starting from 1, which means that this number is the number of each RR interval (RR interval), then we draw the values of RR interval according to the number of intervals.
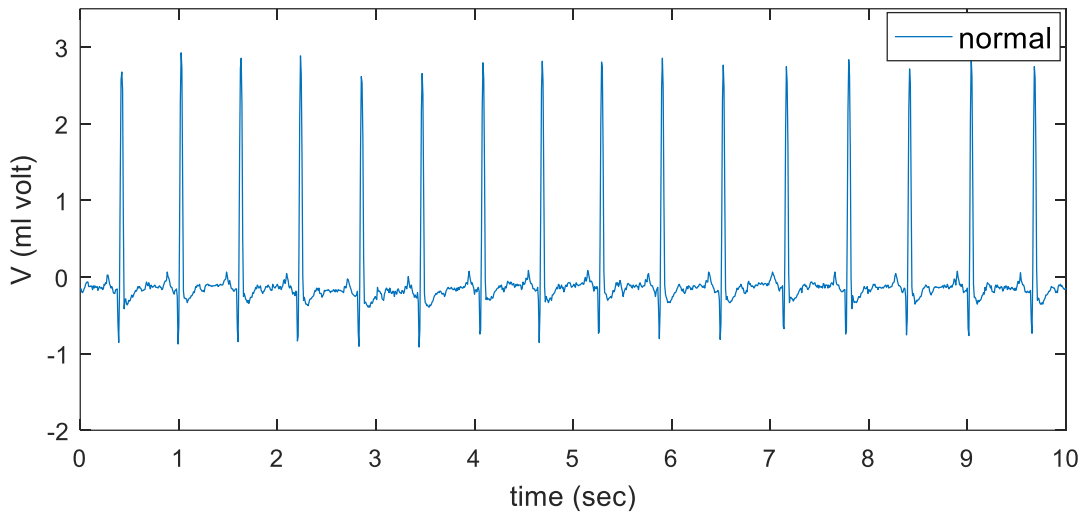

Figure 11: is an ECG diagram of a human with normal heart rhythms.

Figures 11, 13, and 14 show the ECG of a healthy person, a person with ventricular arrhythmia, and a person with atrial fibrillation, respectively. Also, the RR interval series of each of the three mentioned ECGs will be shown in Figures 15, 16, and 17, respectively.
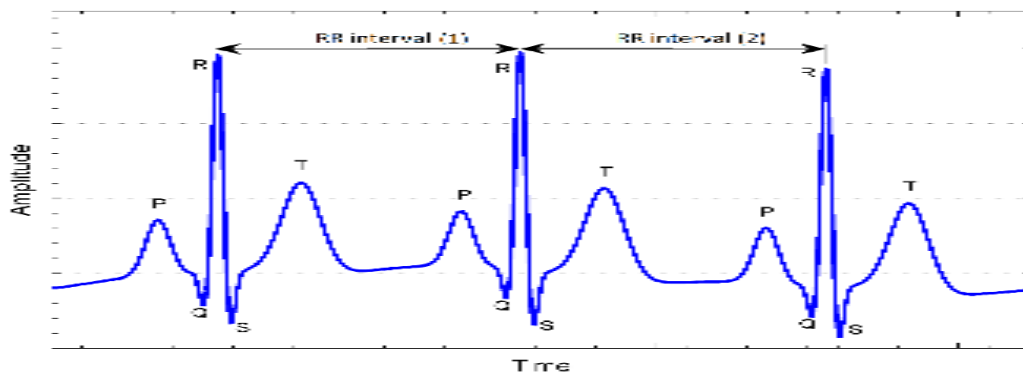
Figure 12: ECG signal components of normal cardiac rhythms (related to humans without cardiac disorders) are named and RR intervals are also shown in it.
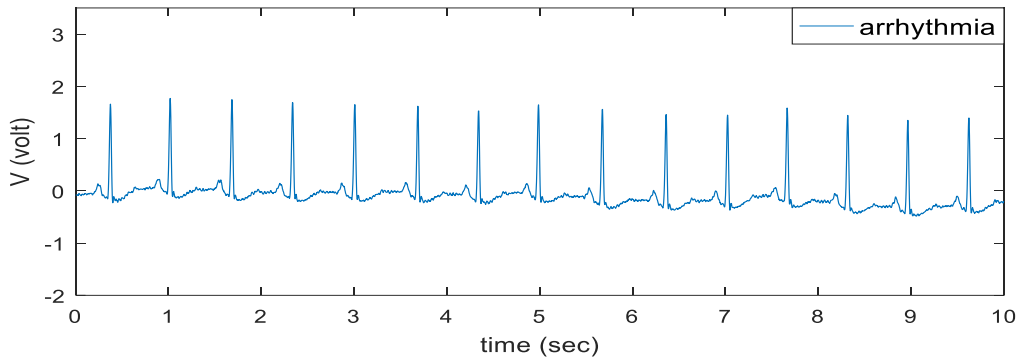

Figure 13: ECG diagram of a human suffering from ventricular arrhythmia.
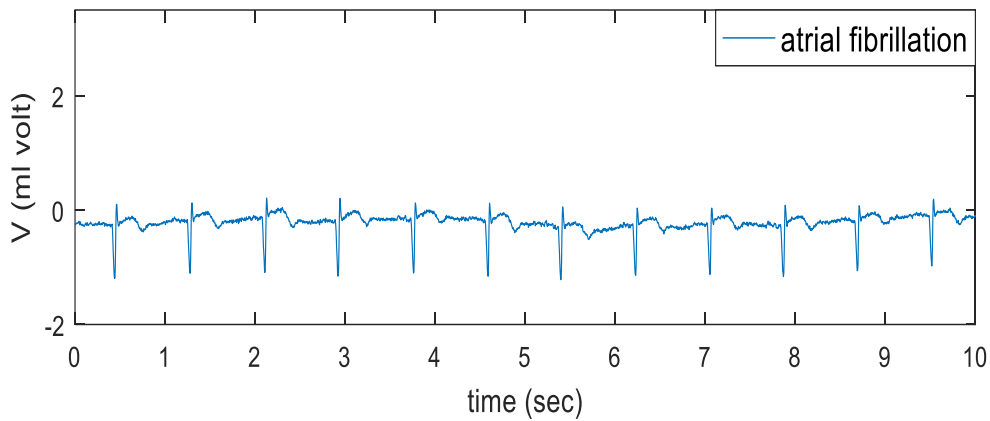

Figure 14: ECG diagram of a human suffering from atrial fibrillation
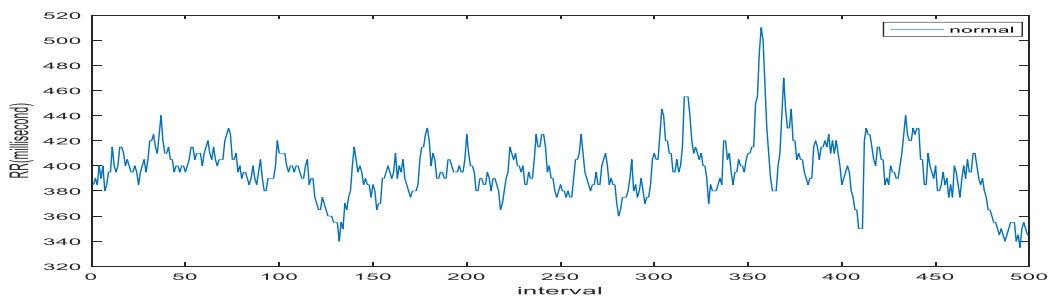

Figure 15: chart of RR interval values related to the ECG of a healthy human (with normal heart rhythm) for each interval.
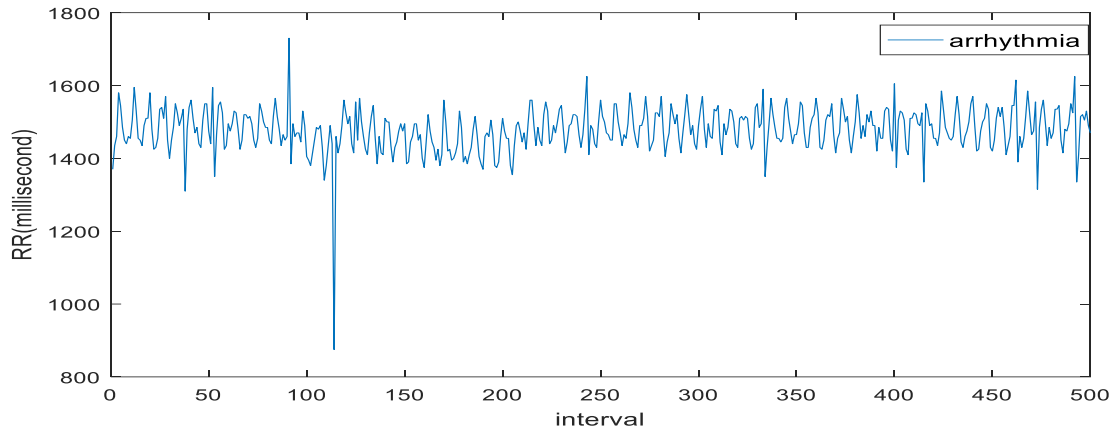
Figure 16: chart of RR interval values related to the ECG of a human with ventricular arrhythmia for each interval.
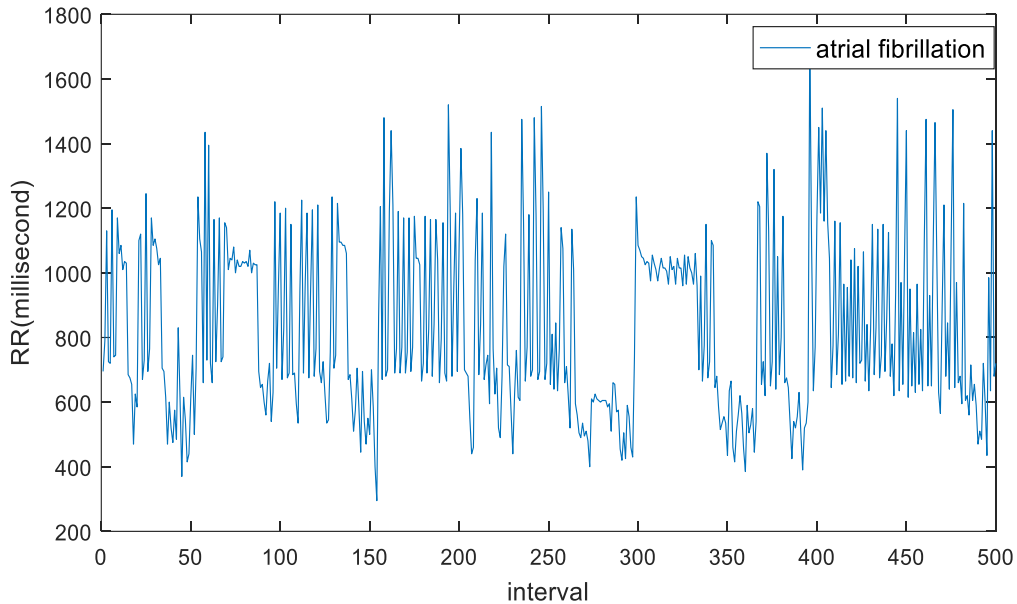

Figure 17: RR interval diagram related to the ECG of a human with atrial fibrillation for each interval.

6. Obtaining the probability density of the extreme values of the R-R distance series using the maximum block method

Therefore, there are three groups of people under study, which include 18 people (without heart disorder), 25 people with atrial fibrillation, and 48 people with ventricular arrhythmia. Having the ECG data of each person, we obtain the RR interval series of each person using the MATLAB program and then find the probability density function of the extreme values of each series using the maximum block approach. Figure 18 Probability density function diagram of the extreme values of the RR interval series obtained by the maximum block method with the series length equal to 1600 and block size 4 (each block contains 4 data from the RR interval series) related to three cases of the subjects under study, one of which It shows that he is healthy and of the other two people, one is suffering from

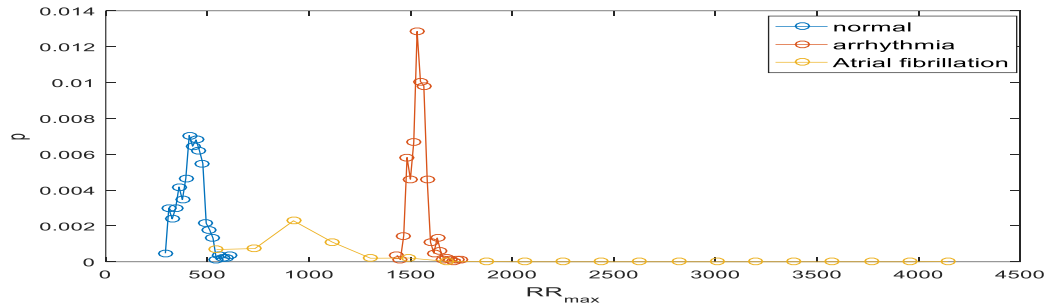ventricular arrhythmia and the other is suffering from atrial fibrillation.

Figure 18: Graphs of probability density functions

Figure 18 Graphs of probability density functions of extreme values of RR distance obtained by maximum block method with block size selection equal to 4 for three subjects under study, one of them is healthy and one of the other two subjects has arrhythmia. Ventricular and the other is suffering from atrial fibrillation.

7. Probability density of the extreme values of the RR distance series by changing the size of the blocks

In this part, we obtain the probability density function of the extreme values of the RR interval series related to each of the three groups of people under study, using the maximum block approach. The diagram of the probability density functions of the extreme values of the RR distance series of all three groups of people under study for choosing the block size equal to 4 is shown in Figure 19. The blue graphs on the left are the probability density function of the extreme values of healthy people, and the red and green graphs correspond to the probability density function of the extreme values of people with atrial fibrillation and ventricular arrhythmia, respectively. We repeated the calculations to find the probability density function of the extreme values of the RR distance series for choosing the size of 16 and 64 for the blocks. The results of repeating our calculations can be seen in Figures 20 and 21, which show the same result as what we got in the case of blocks with sizes equal to 4.
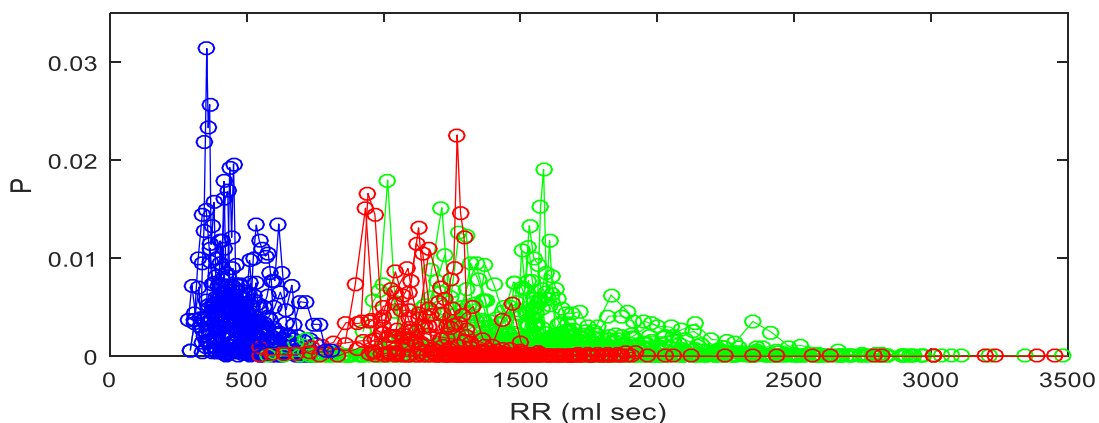
Figure 19: shows the graphs of the probability density functions

Figure (19) shows the graphs of the probability density functions of the extreme values of the RR distance series obtained from the maximum block method for selecting the block size equal to 4 for the subjects under study. The blue graphs are for healthy people and the red and green graphs are for people with atrial fibrillation and ventricular arrhythmia, respectively.
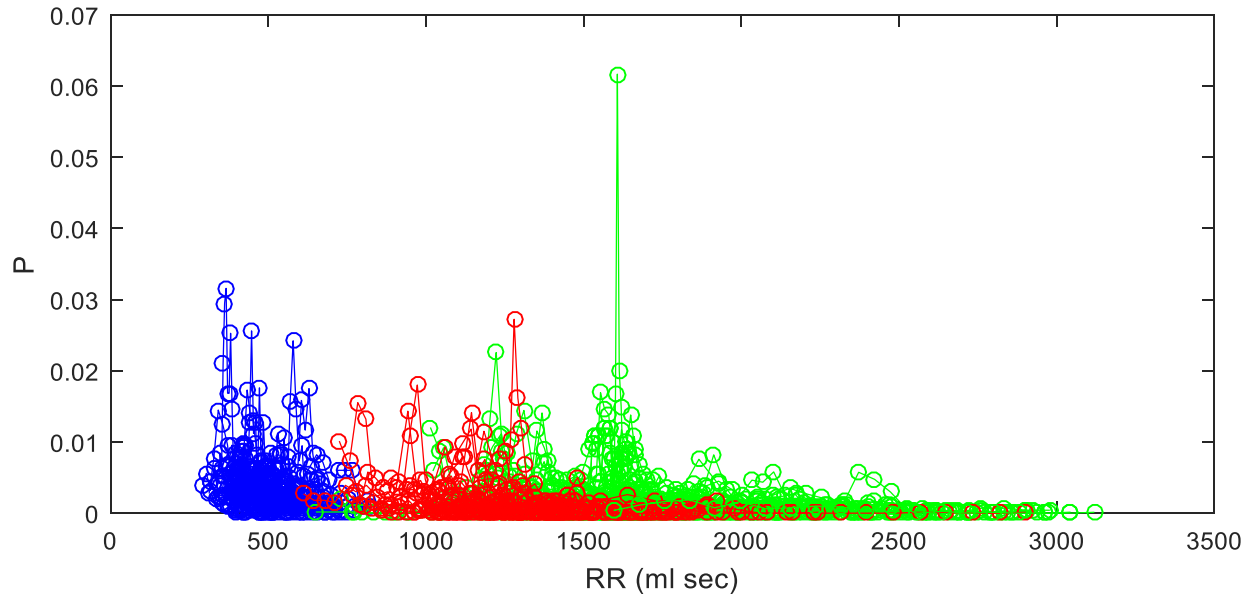
Figure 20: shows the graphs of the probability density functions

Figure (20) shows the graphs of the probability density functions of the extreme values of the RR distance series obtained by the maximum block method for selecting the block size equal to 16 for the subjects under study. The blue graphs are for healthy people and the red and green graphs are for people with atrial fibrillation and ventricular arrhythmia, respectively.

Figure 21: shows the graphs of the probability density functions

Figure (21) shows the graphs of the probability density functions of the extreme values of the RR interval series obtained from the maximum block method for selecting the block size equal to 64 for

the subjects under study. The blue graphs are for healthy people and the red and green graphs are for people with atrial fibrillation and ve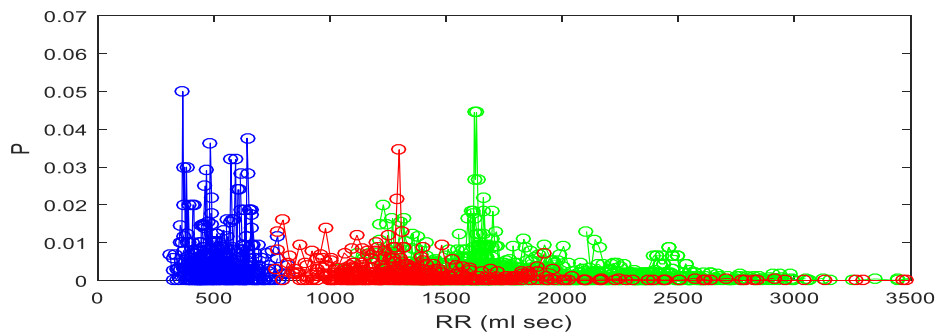ntricular arrhythmia, respectively. As it is clear from all the graphs, there is a significant difference between the distribution of extremes of healthy and sick people. Of course, there is a slight difference between the two groups of people with heart problems. This means that the time series behavior of the heart of a healthy person has less extreme values (average) and also less amount of changes (standard deviation) than the heart of people with heart failure. In other words, this method can be used as a diagnostic method for heart diseases.

8. Obtaining the probability density of the extreme values of the R-R distance series by the method beyond the threshold

In this section, using the approach beyond the threshold, we investigate the behavior of the probability density caused by the extreme values of the RR interval series (which we calculate from the ECG time series) related to healthy people and people with heart disorders. The people under study are considered in three categories. Which were studied in the previous section with the maximum block approach. Figure 22 shows the behavior of the probability density of the extreme values of the RR distance series obtained from the method beyond the threshold for choosing the extreme threshold value of RR equal to 600 milliseconds for three cases of the subjects under study. From the figure, we can see that for the method beyond the threshold, a distinction can be made between a healthy person and two sick people.
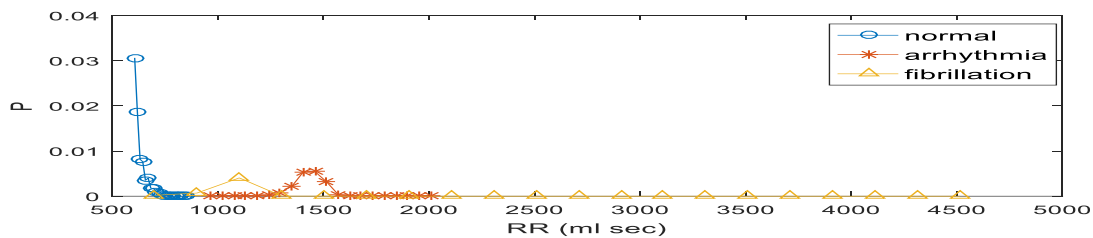

Figure 22: Probability density diagrams

Figure 22 Probability density diagrams of the extreme values of the RR interval series obtained from the method beyond the threshold for the threshold value equal to 600 milliseconds for the RR interval series related to three cases of the subjects under study, one of them without heart disorder (diagram where the points are marked as circles), and from the other two cases, one with ventricular arrhythmia (diagram where the points are marked as stars) and the other with atrial fibrillation (diagram

where the points are marked as stars) triangles are specified).

9. The effect of correlations and non-Gaussian distribution function on the extreme events of the RR distance series

In the following, we will examine the effects of linear and non-linear correlations and Gaussian distribution related to the data of RR interval series on the probability density function of extreme events (extreme values of the RR interval series). For this purpose, we pass the data of the RR distance series under investigation through three filters:

Shuffle, random phase and Ranked wise, and then the probability density function of the extreme events of the resulting series with the maximum block approach We get a separate approach beyond the threshold and describe them to find the difference between the probability density of the extreme values of healthy and sick people. Now we obtain the probability density distribution of the extreme events of the studied RR interval series in the absence of correlations. For this purpose, we consider the RR interval series related to each of the people under study and then we find the combined series corresponding to them and each of the resulting series is an RR' interval series. We call it The probability density function of the extreme values of the RR' distance series obtained from the maximum block method for selecting the block size equal to 4 is shown in Figure 23 and for selecting

the threshold of 600 milliseconds for the over-threshold method in Figure 24. According to Figure 23, by comparing the probability density function of all three categories of people under study, we observe that by using the maximum block approach, maintaining the non-Gaussian distribution function and removing linear and non-linear correlations in the RR distance series. These people, the difference between the probability density of extreme events of healthy and sick people decreases. From the comparison of Figure 23 with Figure 19, we can see that in Figure 23, the probability density functions of the extreme values of healthy people and sick people are closer to each other, so it can be said that there is a distance between the data of each of the series. RR, there are linear and non-linear correlations that affect the behavior of the probability density of the extreme values in them.
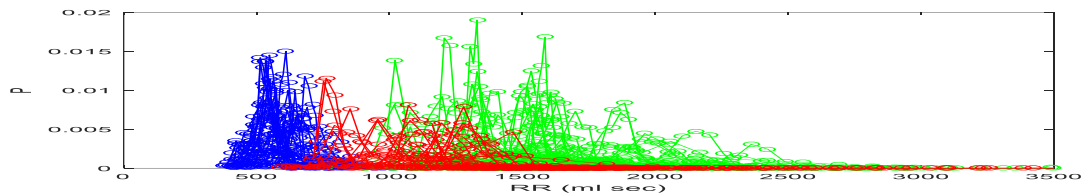

Figure 23: shows the graphs of the probability density functions

Figure 23 shows the graphs of the probability density functions of the extreme values of the distance RR' obtained from the maximum block method for selecting the block size equal to 4 for the

subjects under study. The blue graphs are for healthy people and the red and green graphs are for people with atrial fibrillation and ventricular arrhythmia, respectively.

Figure 24: shows the graphs of the probability

Figure 24 shows the graphs of the probability density functions of the extreme values of the RR' interval obtained from the beyond-the-threshold method for selecting the RR' threshold value equal to 600 milliseconds for the subjects under study. The blue graphs are for healthy people and the red and green graphs are for people with atrial fibrillation and 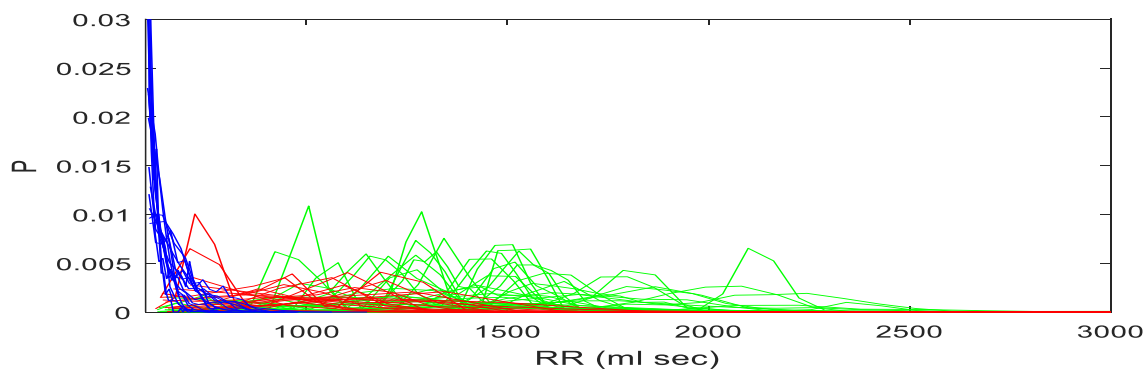ventricular arrhythmia, respectively. Now, we perform the above steps for the RR distance series of the subjects under study so that by maintaining nonlinear and linear correlation, their distribution becomes Gaussian, and we consider each one as RR". Graphs of the probability density functions of the extreme values of the distance series RR", obtained from the maximum block method for choosing the block size equal to 4 in figure (25) and for choosing the threshold of 600 for the method beyond the threshold in figure (26) is shown. From the comparison of Figure 25 and Figure 19, we can see that in Figure 25, the probability density functions of the extreme values of healthy people and sick people are closer to each other, so it can be said that the non-Gaussian distribution function of the RR distance series in differentiation Healthy and sick people have a significant role.
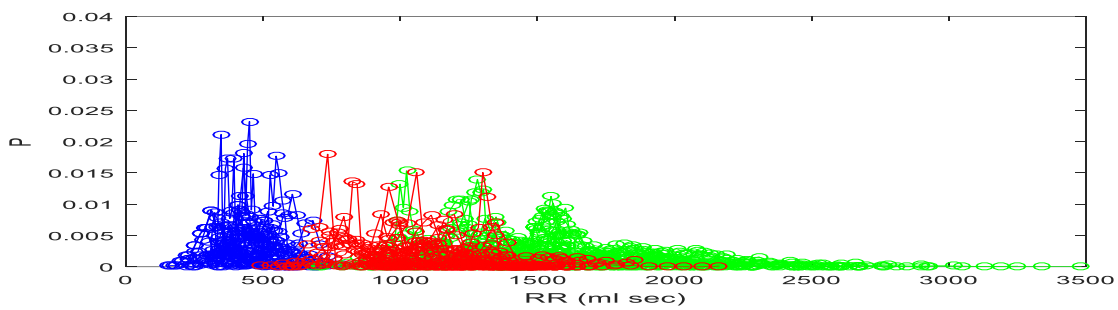

Figure 25: Graphs of the probability density functions

Figure (25) Graphs of the probability density functions of the extreme values of the RR" distance obtained from the maximum block method for selecting the block size equal to 4 for the subjects under study. The blue graphs are for healthy people and the red and green graphs are for people with atrial fibrillation and ventricular arrhythmia, respectively.

Figure 26: shows the graphs of the probability density functions

Figure 26 shows the graphs of the probability density functions of the extreme values of the RR" distance obtained from the beyond-the-threshold method for selecting the RR" threshold equal to 600 milliseconds for the subjects under study. The blue graphs are for healthy people and the red and green graphs are for people with atrial fibrillation an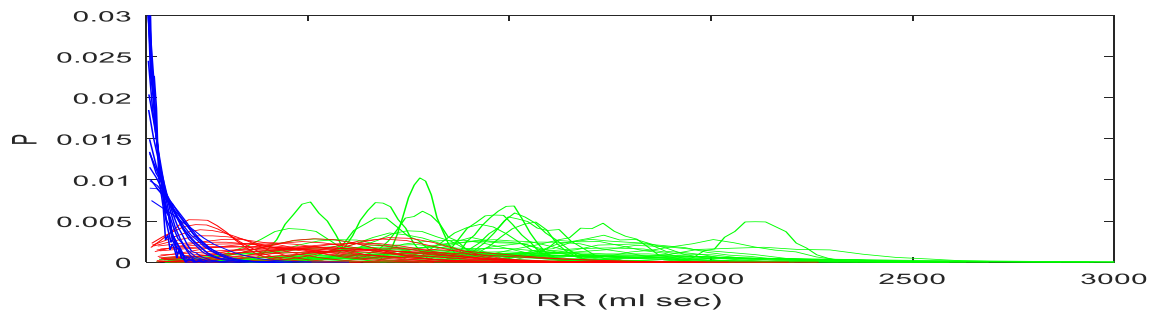d ventricular arrhythmia, respectively. Now, we perform the mentioned steps for the RR distance series of the subjects under study so that they all become nonlinear without correlation, but their linear correlation is maintained and their distribution is Gaussian and each of these new series

We consider it as RR'''. The probability density function of the extreme values of the RR''' distance series obtained from the maximum block method for choosing the block size equal to 4 is shown in Figure 27 and for choosing the threshold of 600 for the method beyond the threshold in Figure 28. From the comparison of Figure 27 with Figure 19, we can see that in Figure 27, the probability density functions of the extreme values of healthy people and sick people are closer to each other, so it can be said that the non-linear correlation of the RR distance series in differentiating people Healthy and sick both play a significant role.
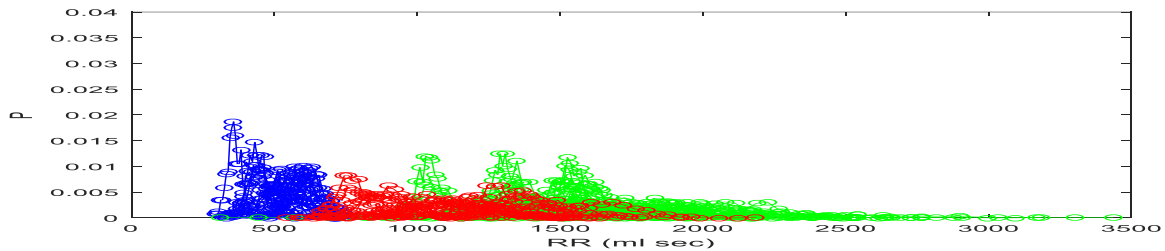

Figure 27: shows the graphs of the probability

Figure 27 shows the graphs of the probability density functions of the extreme values of the RR" distance obtained from the maximum block method for selecting the block size equal to 4 for the subjects under study. The blue graphs are for healthy people and the red and green graphs are for people with atrial fibrillation and ventricular arrhythmia, respectively.

Figure 28: shows the graphs of the probability

Figure 28 shows the graphs of the probability density functions of the extreme values of the RR^" interval obtained from the beyond-the-threshold method for selecting the RR^" threshold equal to

600 milliseconds for the subjects under study. The blue graphs are for healthy people and the red and green graphs are for people with atrial fibrillation and ventricular arrhythmia, respectively.
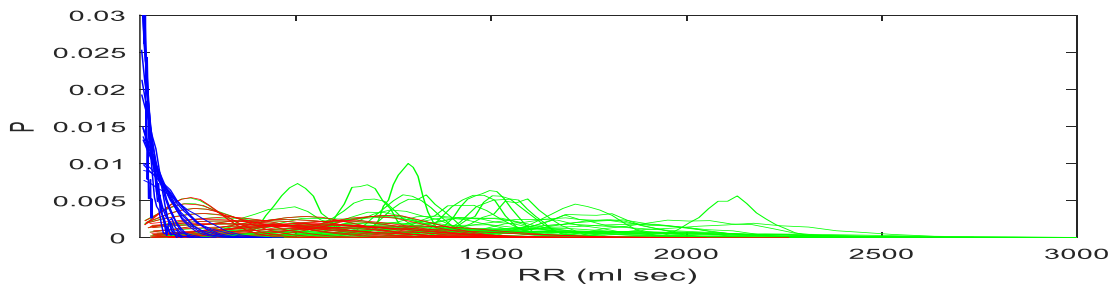
10. Conclusion

In this paper, we considered the RR interval series of three groups of healthy people, those with ventricular arrhythmia, and those with atrial fibrillation, and we tried to find the probability density function of extreme values with the help of the maximum block approach and the beyond-the-threshold approach. Get the RR interval series of each healthy and sick person and distinguish healthy and sick people by the behavior of these functions. First, regardless of the distribution function of the data of each RR series and the correlation between them, with the help of the maximum block approach, we drew the graph of the probability density function of the extreme values of the series of all individuals in one figure. We observed a significant difference from comparing the graphs of healthy people with sick people, in such a way that the probability density of the extreme values of the RR interval series of each healthy person, in the RR interval greater than 900 milliseconds, is zero, but the density functions The probability of extreme values of the RR interval series of each patient has a value or values greater than zero in the RR interval greater than 900 milliseconds. By reaching beyond the threshold for three different thresholds, we calculated the probability density function graph of the extreme values of the RR interval series of each person, but no significant difference between the graphs of healthy and sick people was revealed. To investigate the effect of correlations and distribution function on the distribution of extreme events of the RR distance series of each individual, we first went to a state where, while maintaining the distribution function, all correlations between the data of the series under study are zero. The result of our observations in this case was that by using the maximum block approach and even beyond the threshold, no significant distinction can be found between the probability density functions of the extreme values of the RR interval of healthy and sick people, i.e. linear or non-linear correlation and or both, play a big role in creating extreme events of the RR distance series of the subjects under study. To investigate the effect of the non-Gaussian distribution function alone, we kept the correlation between the data almost unchanged by using the species order method, but we made the distribution function Gaussian, we observed that in this case, the maximum block approach can distinguish the probability density functions. The extreme values of the RR interval of healthy people are not different from those of sick people, but the approach beyond the threshold for choosing the RR interval threshold equal to 600 milliseconds can distinguish between healthy and sick people, so that the lowest value of the RR interval in the distribution of values The extreme RR interval of each patient has a probability density of less than 0.005, but this value is greater than 0.005 for healthy people.

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