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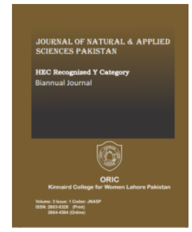

STUDY THE DIFFERENTIAL CROSS SECTION OF E-H ELASTIC SCATTERING IN THE PRESENCE OF LINEAR POLARIZED LASER FIELD

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Abstract

The present work aims to investigate the differential cross section (DCS) of e-H elastic scattering in the presence of a linear polarized laser field for zero to third order Bessel function. A theoretical model is used to study the DCS, which includes the Volkov function, scattering matrix, Coulomb potential, and Born first approximation. The calculated DCS results show interesting features that are analyzed. For zero and first order Bessel function of the first kind, the DCS decreases with momentum transformation from incidence to target. In contrast, the DCS is constant for second order Bessel function of the first kind, while for the third order Bessel function, the DCS increases. These results indicate that the DCS generally decreases with Bessel function of the first kind and momentum transformation. The theoretical model used in this study provides a useful tool to investigate e-H elastic scattering in the presence of a linear polarized laser field. This model can be extended to study more complex systems and can be used to understand the dynamics of electrons interacting with molecules and other materials. The findings presented in this work may have practical applications in fields such as laser physics and materials science. This work provides valuable insights into the behavior of e-H elastic scattering in the presence of a linear polarized laser field. The results highlight the importance of considering the higher-order Bessel functions of the first kind and the role of momentum transformation in the scattering process. The findings presented in this work may have implications for the design of future experiments and the development of new theoretical models for studying electron-molecule interactions.

Keywords
Differential cross section, Linear Polarized laser field, Bessel Function, Born first Approximation

1. Introduction

When two particles that are initially far apart approach one another, interact, and then finally move apart, this phenomenon is known as scattering. J.J. Thompson made the discovery of the electron in 1897 and Rutherford discovered the nucleus in 1911 after the elastic scattering of alpha particles using foil. The discovery additionally sparked the creation of the planetary model of atomic structure. Bohr proposed a model of hydrogen in 1912 that also resulted in the creation of new laws and formulation in the field of physics. Compton conducted a scattering experiment in 1923 that proved photons existed, and Bothe and Geiger corroborated and validated Compton's findings (Bhatia, 2020). One of the simplest atoms to work with, the hydrogen atom can be utilized to gather interesting parts of the issue. This work looks the effects of different collisions and laser variables on the collision process in an elastic electron-hydrogen atom collision aided by a linear polarized laser for zero to third order Bessel function. The energy transfers between electromagnetic radiation and free electrons, which takes the form of diverse processes as bremsstrahlung, Smith-Purcell radiation, Cerenkov radiation, or Compton scattering, is the basis for the time-domain structuring of electron pulses was not study for higher order Bessel function higher-order (first and second) in linear laser field. Additionally, the development of innovative light sources like free electron lasers or high-harmonic generation, as well as ultrafast structural probing techniques like high-harmonic spectroscopy or laser-induced electron diffraction, depend on electron-photon coupling. The first Born differential cross - sectional area for the Mott scattering of a Dirac-Volkov electron, calculated by Szymanowski *et al.,* is reviewed and corrected by Taj *et al.,* in 1997. Tej *et al.,* provide the precise coefficients multiplying the various Bessel functions included in the scattering differential cross section, and they specifically disagree with the formulation. These corrections are crucial because the Dirac-Volkov charged particle's relativistic electronic dressing results in coefficients that multiply the different Bessel functions, and the relativistic research of other processes (like excitation, ionization, etc.) is highly dependent on the accuracy and dependability of the calculations for this procedure of Mott Scattering in the presence of a laser field (Taj *et al.*, 2010). For the dispersion of an electron by the Coulomb potential of a nucleus in the presence of a significant laser field to yield interesting significance and the signatures of the relativistic effects. The Mott scattering process in a strong laser field and revised the Mott scattering differential cross section. The findings of an estimate of the first Born differential cross - sectional area for the Coulomb dispersion of the Dirac-Volkov electrons dressed by a circularly polarized laser field were compared to the first Born cross - sectional area for the Coulomb scattering of spinless Klein-Gordon particles as well as to the non-relativistic Schrodinger-Volkov treatment (Bagrov and Gitman, 1990). The elastic scattering of electrons by hydrogen atoms in the presence of two homogeneous, single-mode, and linearly polarized laser beams has been investigated within the first Born approximation framework. For various geometrical arrangements of the applied laser fields, differential scattering cross-sections for one weak field photon emission have been estimated with regard to incident electron momentum as well as momentum transfer (Bhattacharya *et al.,* 2002). The region of high scattering energy and moderate field intensities for inelastic Electron-H(2s) dispersion in a linearly polarized laser beam. Since it may be challenging to distinguish the signal from elastic and inelastic scattering channels in experimental research, it is crucial to assess the contribution of laser-assisted inelastic electron-atom scattering to the total electron energy spectrum. First, a Gordon-Volkov wave function precisely describes the relationship between both the projectile electron and the laser field. Second, the first-order time-dependent perturbation theories in the field contains a description of how the laser field dresses the hydrogen atom (Buica, 2015). Theoretical research is done on the interaction of an electron with an atom in a field of powerful electromagnetic radiation that is in resonance with two atomic multiplets. Expressions are developed for the photon emission and absorption amplitudes of elastic and inelastic scattering. It is demonstrated that photon emission occurs in transitions from a state of the higher multiplet and photon absorption occurs primarily in the condition of resonance in inelastic transitions from a condition of the lower multiplet (Agre and Rapopor, 1982). In the existence of a resonant laser field, the rotational wave approximation is used to study the scattering of electrons by hydrogen atoms in the metastable 2s state. The $2s^3p$ transition frequency of the hydrogen atom is set to match the frequency of the circularly polarized laser field. It is examined how the cross section varies with laser power (10^6–10^{11}Wcm^2) and input electron energy (50–150 eV). Volkov state can accurately depict the wavefunction of a freely free electron embedded in a laser field, but the proper description of laser modified atomic states remains a major challenge (Purohit *et al.,* 1998). Yadav *et al.,* investigate the circularly polarized (CP) laser field's effects on the scattering of an electron by hydrogen atoms and found that the differential scattering cross - sectional area reduces with an increase in scattering angle. Finally, it was found that the polarization of the laser field has a significant impact on the differential scattering cross section (Yadav *et al.,* 2017). In addition, Yadav *et al.,* also study the DCS in presence of elliptical polarized laser field with electron and hydrogen. It was found that DCS increase as the incident electron's kinetic energy rises, while altering the value of the polarizing angle has no impact on DCS with kinetic energy (Yadav *et al.*, 2020). The thermodynamic properties of thermal electrons participating in scattering events was studies by (Dhobi *et al.*,) and study shows thermodynamic energy, around the target with distance at field amplitude 0.1 a.u. to 0.9 a.u. has destructive interference, above field amplitude 1 a.u. to 2.5 a.u. has superposition and at field amplitude 2.5 a.u. and 3 a.u. have coulomb potential like nature. Also, thermodynamic energy with temperature was found constant except at field amplitude 2.5 a.u. and at field amplitude 2.5 a.u. destructive interference at 10 ^{o}C and 21 ^{o}C. The thermodynamical potential at field amplitude 0.1 to 3.5 a.u. found constant and above field amplitude 3.5 a.u. increased linearly when studied with respect to temperature at 10 Å. The thermal Hamiltonian increase sharply when thermal electrons enter in 1 to 5 Å, slowly in 5 to 10 Å and beyond 10 Å constant, and the thermal Hamiltonian nature is like coulomb potential (Dhobi *et al.,* 2022). Strong radiation fields exposed to atomic matter give rise to a vast area of current theoretical and experimental investigation. The detection of multiphoton processes has been made possible by the advent of powerful, adjustable lasers at relatively low light field strengths. Comparatively speaking to the difficulties involved in the consideration of field free electron-atom scattering, the theoretical analysis of electron-atom collisions in the presence of a laser field becomes exceedingly complex (Mason, 1993). (Dhobi *et al.,)* investigate the DCS in the presence of a low laser field (visible and UV) and predicts that DCS increase with absorbs of energies. In addition, DCS with emits energy from target found DCS initially drops to a minimum and reaches to maximum. At 5 eV, 10 eV, 13 eV, 16 eV, 20 eV, 25 eV, and 30 eV when energy is emitted by target. Also, the DCS increase as the scattering angle increases (Dhobi *et al.,* 2022). In presence of elliptically polarized beam, the DCS increase with wavelength and become low with electron energy. At 1.56 radian polarized angle, the DCS reaches its maximum value, and at -1.56 radian polarized angle, its minimum value, in energy range 0 to 600 eV electron energy with 1.5eV laser photon energy at 10^{14} Wcm^2 laser field strength (Yadav *et al.,* 2021). The idea of a cross section, which may be described by theories and tested by experiments, is used to quantify scattering theories and experiments. A projectile will be deflected or scattered when it approaches a target because of the force the object applies to it. It is impractical to try to quantify each contact a projectile has with a target. The interaction region is defined as the space between the projectile ray and the target assembly. The projectile has two outcomes at the interaction region: it can hit a target and disperse, or it can miss. The relationship between the projectile-target interaction and some physical property, such as the direction of projectile scattering. The region of interaction between projected-target, total cross section over all solid angles is defined as

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = \int \frac{d\sigma}{d\Omega}(\theta,\varphi)\, sin\theta\, d\theta\, d\varphi \quad (1)$$

In addition, since we consider linear polarization laser field we have vector potential:

$$\boldsymbol{A} = \boldsymbol{a}\, cos(\omega t) \quad (2)$$

where **a** is vector potential amplitude and according to Bransden & Joachain, cross sections, the text Modern Quantum Mechanics has the following derivation of the cross section from the transition rate (Sakurai and Napolitano, 2011) and the quantum scattering theory's transition rate is given by

$$w(i \rightarrow f) = \frac{mk_f(2\pi)^3}{(2\pi)^2\hbar^3}\left|T_{fi}\right|^2 d\Omega \quad (3)$$

where T_{fi} is the transition matrix, m is scattering particle's mass, dΩ is solid angle m and k_f is final momentum. The incident flux and differential cross section is obtained as

$$\frac{d\sigma}{d\Omega} = \frac{k_f}{k_i}\left(\frac{m(2\pi)^3}{(2\pi)\hbar^2}\right)^2 |\,T_{fi}\,|^2 \quad (4)$$

2. Methods and Materials

2.1 Volkov Wavefunction

The time-dependent Schrodinger equation (TDSE) is used to determine the wave equation of an electron associated with an external electromagnetic field (Kim, 2022) in equation (6), vector potential **A**(t) the Hamiltonian of an electron connected to an external electromagnetic field is obtained as

$$H_{e-laser} = \frac{1}{2m}[\hat{p} - eA(t)]^2 \qquad (5)$$

The TDSE

$$i\hbar\frac{\partial}{\partial t}X(r,t) = \frac{1}{2m}[\hat{p} - eA(t)]^2 X(r,t) (6)$$

Atomic physics textbooks like the one by can be used to find the answer to equation (6) (Bransden and Joachain, 2003) the TDSE wave function as

$$X(r,t) = 1\frac{1}{(2\pi)^{3/2}} exp\{i\frac{p}{h} \cdot (r + \frac{e}{m}\int A(t')dt' - i\frac{E}{\hbar}t - i\frac{e^2}{2m\hbar}\int A^2(t')dt'\} \quad (7)$$

This equation (7) is also known as Volkov wave function and with vector potential for linear polarized field as

$$X(r,t) = \frac{1}{(2\pi)^{\frac{3}{2}}} exp \left\{- \frac{i}{\hbar}\left(E + \frac{e^2a^2}{4m}\right)t + i\frac{\boldsymbol{p}}{\hbar} \cdot \left(\boldsymbol{r}\frac{e\,\boldsymbol{a}}{m\omega} sin(\omega t)\right) - i\frac{e^2a^2}{8m\hbar\omega} sin(2\omega t)\right\} \qquad (8)$$

Now, according Kroll Watson Approximation (KWA) the scattering matrix with potential and wave function is obtained by (Kroll and Watson, 1973) as,

$$S_{fi} = = \delta_{fi} - \frac{i}{\hbar}\int_{-\infty}^{+\infty}\langle X_f(r,t)|V(r)|\Psi_i(r,t)\rangle dt' \quad (9)$$

Now from equation (9) the time integral by supposing that the scattered potential is low (Griffiths, 2004),

$$\frac{i}{\hbar}\int_{-\infty}^{+\infty}\langle X_f(r,t)|V(r)|X_i(r,t)\rangle dt' \quad (10)$$

In addition, we get, the matrix element as

$$\int_{-\infty}^{+\infty}\langle X_f(r,t)|V(r)|X_i(r,t)\rangle dt' = \frac{1}{(2\pi)^3}\frac{i}{\hbar}\int_{-\infty}^{+\infty} e^{i(E_f - E_i)t'/\hbar}dt' \times \int_{-\infty}^{+\infty} exp\{-i\frac{e}{m\hbar\omega}\boldsymbol{Q}.\boldsymbol{a}\,sin(\omega t)\}dt' \times \int V(\boldsymbol{r})e^{-i(\boldsymbol{Q}\,.\,\boldsymbol{r})/\hbar}d^3 r' \quad (11)$$

Where momentum transfer formula $\boldsymbol{Q} = \boldsymbol{p_f} - \boldsymbol{p_i}$and on applying Jacobi-Anger Expansion with scattering amplitude of the first born approximation (Sakurai and Napolitano, 2011) and Function's inverse Fourier transform (Boas, 2006) we get,

$$\frac{i}{\hbar}\int_{-\infty}^{+\infty}\langle X_f(r,t)|V(r)|X_i(r,t)\rangle dt' = -\frac{1}{(2\pi)^3}\frac{i}{\hbar}\frac{2\pi\hbar^2}{m}\sum_{n=-\infty}^{+\infty}J_n\left(-\frac{e}{m\hbar\omega}(\boldsymbol{Q}\,.\,\boldsymbol{a})\right) - \frac{me^2}{\hbar^2Q^2}\int_{-\infty}^{+\infty}e^{\frac{i(E_f - E_i + n\hbar\omega)}{\hbar}}dt' = -2\pi\frac{1}{(2\pi)^3}\frac{i}{\hbar}\frac{2\pi\hbar^2}{m}\sum_{n=-\infty}^{+\infty}J_n\left(-\frac{e}{m\hbar\omega}(\boldsymbol{Q}\,.\,\boldsymbol{a})\right) \times \left(-\frac{me^2}{\hbar^2Q^2}\right)\delta\left(\frac{E_f - E_i + n\hbar\omega}{\hbar}\right) \quad (12)$$

Now, the S matrix can be expressed from above for the n^{th} element.

$$S_{fi} = \delta_{fi} - i2\pi\delta(E_f - E_i + n\hbar\omega) \times \left[-\frac{1}{(2\pi)^3}\frac{2\pi\hbar^2}{m}J_n\left(-\frac{e}{m\hbar\omega}(\boldsymbol{Q}\,.\,\boldsymbol{a})\right)f_{Born}^1\right] \quad (13)$$

Also, the T_{fi} matrix can obtained as

$$T_{fi} = -\frac{1}{(2\pi)^3}\frac{2\pi\hbar^2}{m}J_n\left(-\frac{e}{m\hbar\omega}(\boldsymbol{Q}\,.\,\boldsymbol{a})\right)\left(-\frac{me^2}{\hbar^2 Q^2}\right) \quad (14)$$

Now, the DCS for scattering in atomic unit can be obtained as

$$\frac{d\sigma_{FF}^{n}}{d\Omega} = \frac{k_f}{k_i}\frac{1}{(k_f - k_i)^4}J_n^2\left(-\frac{Qacos\theta}{\omega}\right) \quad (15)$$

Where J is Bessel function defined (Mhtlab, 2023) as

$$J_n = \frac{1}{\Gamma(1+n)}\left(\frac{x}{2}\right)^n\left\{1 - \frac{\left(\frac{x^2}{2}\right)}{1(1+n)}\left(1 - \frac{\left(\frac{x^2}{2}\right)}{2(2+n)}\right)\left(1 - \frac{\left(\frac{x^2}{2}\right)}{3(3+n)}\right)(1 - \cdots \ldots\ldots)\right\} \quad (16)$$

Now for Bessel function order $n = 0$, 1, 2 and 3, $J_0 = 1, J_1 = -\frac{Qacos\theta}{2\omega}$, $J_2 = \frac{Q^2a^2\cos^2\theta}{8\omega^2}$ and $J_3 = -\frac{Q^3a^3\cos^3\theta}{48\omega^3}$, respectively. The DCS for Bessel function order $n = 0$, 1, 2 and 3 is obtained as (17), (18), (19) and (20) respectively as the DCS for $n = 0$ order Bessel function

$$\left(\frac{d\sigma}{d\Omega}\right)_{J_0} = \frac{k_f}{k_i}\frac{1}{(k_f - k_i)^4} \quad (17)$$

Now the DCS for $n = 1$, order Bessel function obtained as

$$\left(\frac{d\sigma}{d\Omega}\right)_{J_1} = \frac{k_f}{k_i}\frac{a^2cos^2\theta}{(k_f - k_i)^2 4\omega^2} \quad (18)$$

Also, the DCS for $n = 2$, order Bessel function obtained as

$$\left(\frac{d\sigma}{d\Omega}\right)_{J_2} = \frac{k_f}{k_i}\frac{a^4\cos^4\theta}{64\omega^4} \quad (19)$$

The DCS for $n = 3$, order Bessel function obtained as

$$\left(\frac{d\sigma}{d\Omega}\right)_{J_3} = \frac{k_f}{k_i}\frac{Q^2a^6\cos^6\theta}{2304\omega^6} \quad (20)$$

3. Results and Discussion

The developed equation (15) is use to calculated the DCS with different order of Bessel function as shown in equation, (17), (18), (19) and (20). To study the DCS with momentum transformation with consider using momentum of incidence electron 0.9eV for elastic scattering in laser field of photon energy at vector potential 1 a.u. (where $(1\,a.u. = 27.21eV)$. Laser-assisted electron scattering (LAES) is a fundamental three body interaction process that enables energy transfer between electrons and photons in the presence of matter with linear polarized laser field between Electron-Helium, for zero order Bessel function. On applying the Kroll–Watson approximation for LAES are in agreement with experimental spectra and yield a mechanistic description of electron generation and the LAES energy modulation processes. The nature of DCS with energy transformation has same nature with authors found in this present work of figure 1 (Treiber *et al.*, 2021). In addition, literature so that limitation of DCS with higher order Bessel function for polarized laser field.

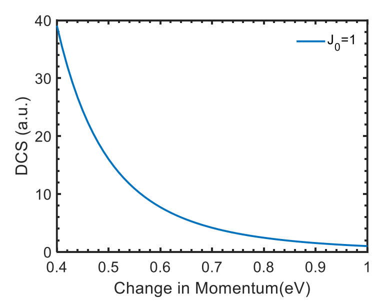

Figure 1: DCS with change in momentum of incidence electron in linear polarized laser field with zero order Bessel function

The DCS with change in momentum of incidence electron decrease for zero order Bessel function as shown in figure 1 is similar to figure 3 (Yadav *et al.*, ., 2017). But they calculated for circular polarized laser beam but in this work a linear polarized beam of laser is used. The different is that DCS for zero order Bessel function found in linear case found greater than circular polarized with very lower transformation of momentum of initial electron in laser beam without absorption and emission, from and to, laser field. Electron-atom potential scattering assisted by a dichromatic (two-component) elliptically polarized laser field is analyzed in the frame of the S-matrix theory. The second Born approximation is applied in the expansion of the S-matrix element. The first term in the expansion corresponds to the single scattering, while the second term in the expansion corresponds to the double scattering of electrons on atomic targets. The double scattering is possible in the presence of a laser field. The electron that has scattered on an atomic target may be driven back by the laser field and scatter again on the same atom but no driven back of electron is observed in linear polarized field laser scattering as shown in figure 1 and figure 2.

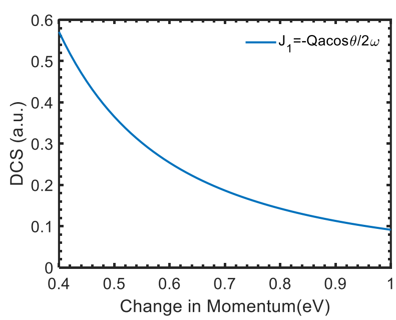

Figure 2: DCS with change in momentum of incidence electron in linear polarized laser field with first order Bessel function

The DCS nature with transformation of momentum with first order Bessel function is show in figure 2. The nature is same is that of zero order but the DCS at zero order found greater than first order Bessel function i.e. $\left(\frac{d\sigma}{d\Omega}\right)_{J_0} > \left(\frac{d\sigma}{d\Omega}\right)_{J_1}$.This shows that the measurement of DCS is vey complex and difficult in compare to zero order. The DCS of second order Bessel function doesn't depend of upon the momentum's transformation, hence it is constant and found 0.002 a.u. This show the DCS of Bessel second order is lower than zero and first order and hence it is complex and difficulties on measurement.

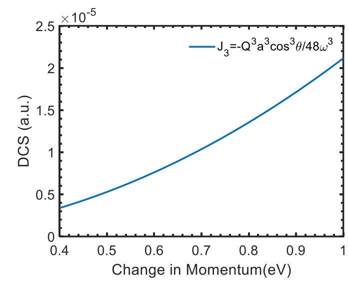

Figure 3: DCS with change in momentum of incidence electron in linear polarized laser field with third order Bessel function

The DCS with momentum transformation is increase but has limit that is this nature was obtained up to 0.002 a.u. (second order Bessel function DCS). If we compare the DCS the DCS with increasing order found decrease with momentum. Therefore, the DCS was found of order $\left(\frac{d\sigma}{d\Omega}\right)_{J_0} > \left(\frac{d\sigma}{d\Omega}\right)_{J_1} > \left(\frac{d\sigma}{d\Omega}\right)_{J_3}$. This shows the measure ment of DCS is complex and difficult at higher order Bessel function because e the measurement region for higher order Bessel function is smaller than lower. The DCS is lower means the momentum transformation of incidence electron is lower therefore we get less information exchanges between the target and incidence particles Here order of Bessel function with DCS means the region where the interaction of particles (target and projected) take place. The increasing the order of Bessel function means electrode are more closure to target and the interaction is very low due to this the DCS of higher order Bessel function is lower. Hence, zero order Bessel function is best for calculating the DCS of scattering in presence of linear polarized laser beam in elastic scattering. The DCS nature with change in momentum decrease as shown in figure 1 and 2 for zero and first order Bessel function, respectively. But the DCS for third order Bessel function found increase this is because with increasing the Bessel function the incidence electron goes closure to nucleus upto certain range and then get repulse by particle formation of same charge insider nucleus (Pi-meson particles). This repulsive nature is not seen on zero and first order because the incidence electron is far away from nucleus and not gets affected by particle (Pi-meson) of same charge from inside nucleus. Understanding laser-assisted electron scattering, dynamics in strong fields, laser-induced alignment, and electron control through the study of e-H elastic scattering in a polarized laser field. It improves our knowledge of fundamental physics, molecular alignment, extreme electron behavior, and laser-induced activities. Asymmetry/interference effects, higher-order scattering processes, laser-induced multiphoton processes, and non-perturbative interactions can all be seen when using higher-order Bessel functions. These applications contribute to our understanding of laser-matter interactions by having consequences for atomic physics, laser spectroscopy, and quantum optics.

4. Conclusion

On developing the equation for DCS in presence of linear polarized laser field. It is found the that DCS with momentum transformation goes decrease upto second order Bessel function but for third order Bessel function the DCS with momentum transformation goes increase. In addition, the DCS for lower order Bessel function was found higher than higher order Bessel for elastic scattering in linear laser field. The DCS, with lower order Bessel function is better than DCS of higher order Bessel function because the higher DCS is obtained at lower order. The higher DCS is better for measurement and remove the complexity on measurement. In addition, the DCS is also known as reaction region where different information gets exchange between target and projected particles. Therefore, to study the information between them the DCS should be higher because in smaller region of reaction to gather the information between particles is difficulties.

References

Agre, M. Y., & Rapoport, L. P. (1982). Electron scattering by an atom in the field of resonant laser radiation. Soviet Physics JETP, 55(2), 215-216.

Bhatia, A. K. (2020). Scattering and its applications to various atomic processes: Elastic scattering, resonances, photo absorption, Rydberg states, and opacity of the atmosphere of the Sun and stellar objects. Atoms, 8(4), 1-30. https://doi.org/10.3390/atoms8040078

Bagrov, V. G., & Gitman, D. M. (1990). Exact solutions of relativistic wave equations. Kluwer Academic Publishers, Dordrecht.

Bhattacharya, M., Malakar, B., & Sarkar, S. (2002). Scattering of electron by hydrogen atom in the presence of two laser fields. Physica Scripta, 66(3), 1-2.

Buica, G. (2015). Laser-assisted inelastic electron scattering by excited hydrogen atoms. In XXIX International Conference on Photonic, Electronic, and Atomic Collisions (ICPEAC2015), *Journal of Physics: Conference Series, 635(092063), 1-2. https://doi.org/10.1088/1742-6596/635/9/092063*

Bransden, B. H., & Joachain, C. J. (2003). Physics of Atoms and Molecules (2nd ed.). Prentice Hall, Harlow, England.

Boas, M. L. (2006). Mathematical Methods in the Physical Sciences. John Wiley & Sons, Inc., Hoboken, NJ.

Dhobi, S. H., Yadav, K., Gupta, S. P., Nakarmi, J. J., & Koirala, B. (2022). Differential cross-section in the presence of a weak laser field for inelastic scattering. Ukrainian *Journal of Physics, 67(4), 227-228. https://dx.doi.org/10.15407/ujpe67.4.227*

Dhobi, S. H., Nakarmi, J. J., Yadav, K., Gupta, S. P., Koirala, B., & Shah, A. K. (2022). Study of thermodynamics of a thermal electron in scattering. Heliyon, 8, 1-2. https://doi.org/10.1016/j.heliyon.2022.e12315

Griffiths, D. J. (2004). Introduction to Quantum Mechanics (2nd ed.). Pearson.

Kim, B. N. (2022). Angular distribution of electron-helium scattering in the presence of a 1.17 eV laser field (Doctoral dissertation). Theses and Dissertations--Physics and Astronomy. No.95. https://uknowledge.uky.edu/physastron_etds/95

Kroll, N. M., & Watson, K. M. (1973). Charged-particle scattering in the presence of a strong electromagnetic wave. *Physical Review* A, 8, 804–809. https://doi.org/10.1103/PhysRevA.8.804

Mason, N. (1993). Laser-assisted electron-atom collisions. Reports on Progress in Physics, 56(10), 1275-1280.

Mhtlab. (n.d.). Bessel functions of the first and second kind. Retrieved from http://www.mhtlab.uwaterloo.ca/courses/me755/web_chap4.pdf

Purohit, S. P., Jain, A. K., & Mathur, K. C. (1998). Elastic scattering of electrons by metastable hydrogen atoms in the presence of a resonant laser field. European Physics *Journal D, 2, 41–44. https://link.springer.com/article/10.1007/s100500050088*

Sakurai, J. J., & Napolitano, J. (2011). Modern Quantum Mechanics. Addison, Wesley, San Francisco, CA.

Taj, S., Manaut, B., Attaourti, Y., Elhandi, S., & Oufni, L. (2010). Coulomb scattering of an electron in strong laser fields. M. J. Condensed Matter, 12(3), 1-2. https://www.researchgate.net/publication/278786092_Coulomb_Scattering_Of_An_Electron_In_Strong_Laser_Fields

Treiber, L., Kanya, R., Zeiler, M. K., & Koch, M. (2022). Dynamics of above-threshold ionization and laser-assisted electron scattering inside helium nanodroplets. *The Journal of Physical Chemistry A, 126(45), 8380–8385. https://doi.org/10.1021/acs.jpca.2c05410*

Yadav, K., Nakarmi, J. J., & Maharjan, S. (2017). Free scattering theory in circularly polarized laser field. *Journal of Nepal Physical Society, 4(1), 85-86. https://npsjournal.nps.org.np/2020/08/07/free-scattering-theory-in-circularly-polarized-laser-field/*

Yadav, K., Gupta, S., & Nakarmi, J. (2020). Elliptically polarized laser-assisted elastic electron-hydrogen atom collision and differential scattering cross-section. *Himalayan Physics*, 9(1), 93–102. https://doi.org/10.3126/hp.v9i01.40205

Yadav, K., Dhobi, S. H., Maharajan, S., Gupta, S. P., Karki, B., & Nakarmi, J. J. (2021). Elliptically polarized laser-assisted elastic electron-hydrogen atom collision in Coulomb potential. *Eurasian Physical Technical Journal, 18(4), 82-84. https://dx.doi.org/10.31489/2021No4/82-87*